Introduction to Petri Nets

2006/2007

History

- Petri nets were introduced by C.A. Petri in his Ph.D. Dissertation: ”Kommunikation mit Automaten.” , Institut für Instrumentelle Mathematik, Bonn, 1962.
- They are particularly useful form modeling systems with concurrent and asynchronous processing

Why concurrency is a problem?

- Concurrency and asynchronous processing is typical in real world
- It can pose a problem when many entities (people, machines, processing threads) use (share) the same resource (or a limited number of resources)
- A trivial example is that of an elevator – the cabin is single resource that many people want to use. The problem is how to control the elevator to minimise waiting time?

Why concurrency is a problem?

- The elevator '”scheduling” control is not crucial, at worst improper algorithm may result in long waiting times
- There are situations where handling concurrency is crucial for correct behaviour of the system

Why concurrency is a problem?

- Unwanted results of concurrency include:
  - Race conditions
  - Resource starvation
  - Deadlocks

Race conditions

- When race conditions occur, the result of the system behaviour may be unexpected and is dependent on the sequence of other events
- Race conditions were first described in electronics (logic circuits), when parallelism is typical
- An example of race conditions may be poorly implemented ATM

ATM case

- Lets imagine the ATM operation is following
  - Authorise client
  - Get account balance
  - Get client request
  - Update account
  - Dispense cash

ATM case

- This can translate to following situation:
  - Time = t_0 Authorisation OK
  - Time = t_0+2s Balance is 1000
  - Time = t_0+10s Client requested 800, 800<1000
  - Time = t_0+12s Account updated by -800, 200 left
  - Time = t_0+14s Cash dispensed
- Everything worked fine

ATM case

- Now, lets imagine that there are two cards attached to the same account (wife and husband, corporate account etc.)
- Two persons at nearly the same time want to withdraw money. What may happen?

ATM case

	Client 1	Client 2
Time = t_0	Authorisation OK	
Time = t_0+1s		Authorisation OK
Time = t_0+2s	Balance is 1000	
Time = t_0+3s		Balance is 1000
Time = t_0+5s		Client requested 800, 800<1000
Time = t_0+7s		Account updated by -800, 200 left
Time = t_0+9s		Cash dispensed
Time = t_0+10s	Client requested 800, 800<1000	
Time = t_0+12s	Account updated by -800, **-600 left**	
Time = t_0+14s	Cash dispensed	

Race conditions

- Race conditions may be resolved by resource locking

	Client 1	Client 2
Time = t_0+2s	Balance is 1000, lock account	
Time = t_0+3s		account locked - cannot proceed
Time = t_0+10s	Client requested 800, 800<1000	
Time = t_0+12s	Account updated by -800, 200 left, unlock account	
Time = t_0+14s	Cash dispensed	

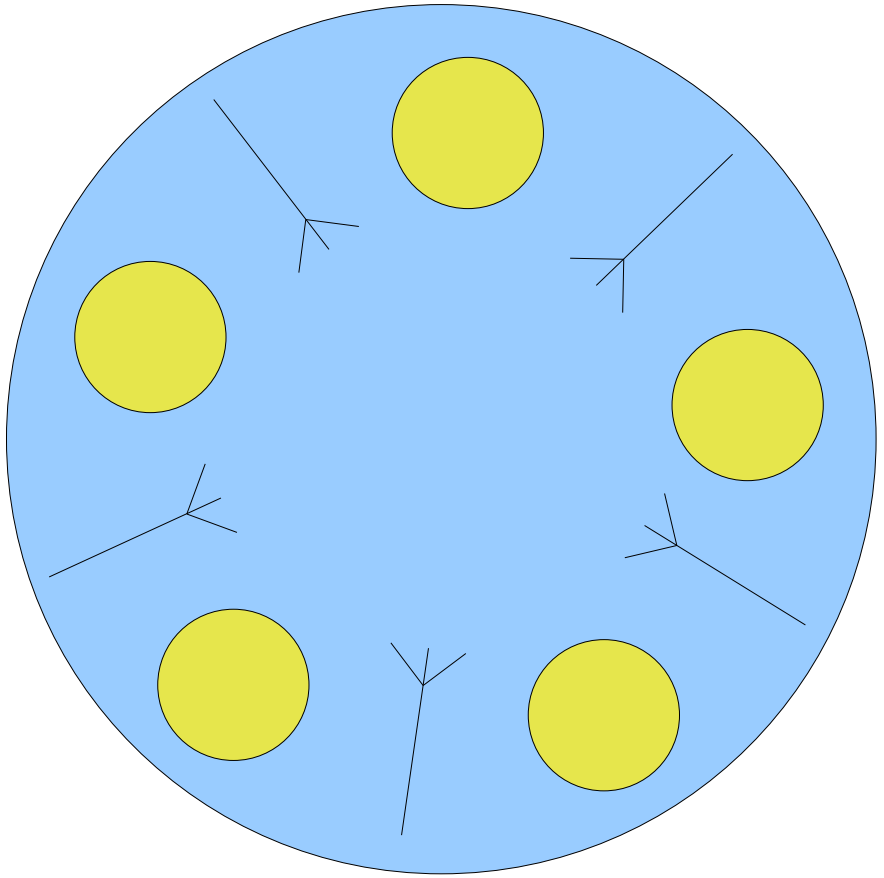
Resource starvation

- The process (person, system) is denied the resource it needs, because other process is not freeing them

Deadlock

- Deadlock occurs when a number (at least 2) processes are waiting for the other to finish (or release resources) and therefore none progresses
- This can be illustrated by dining philosophers problem (invented by Edsger Dijkstra)

Dining philosophers problem

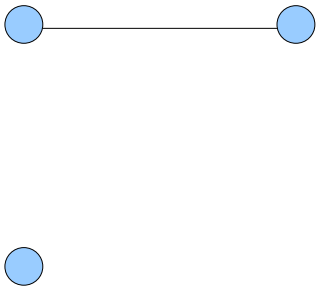


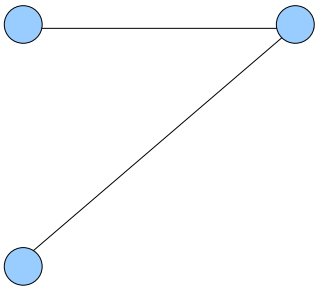
- The philosopher is either thinking, or eating
- The philosophers do not talk (communicate)
- The philosopher can pick a fork to her/his right or left, one at a time
- The philosopher needs both forks to eat

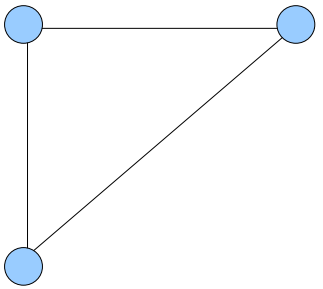
Graphs

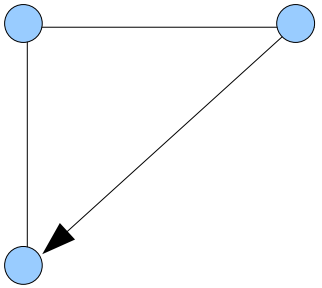
- Petri nets are graphs
- Graph is an ordered pair G:=(V,E), where V is a set of nodes (vertices), E is a set of pairs of distinct vertices – edges (lines)
- If E is a set of unordered vertices, the graph is undirected
- If E is a set of ordered vertices, the graph is directed (digraph)
- Petri nets are digraphs

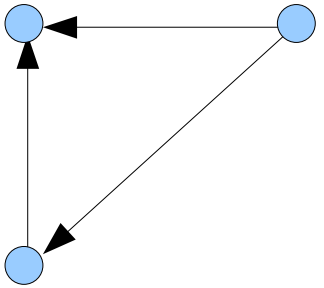
Graphs





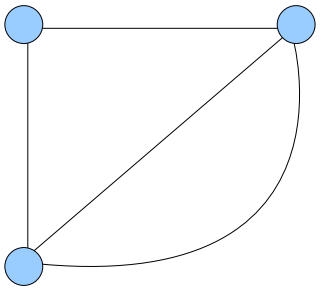






Multigraph

- A multigraph is a graph that may contain more than one edge connecting the same vertices



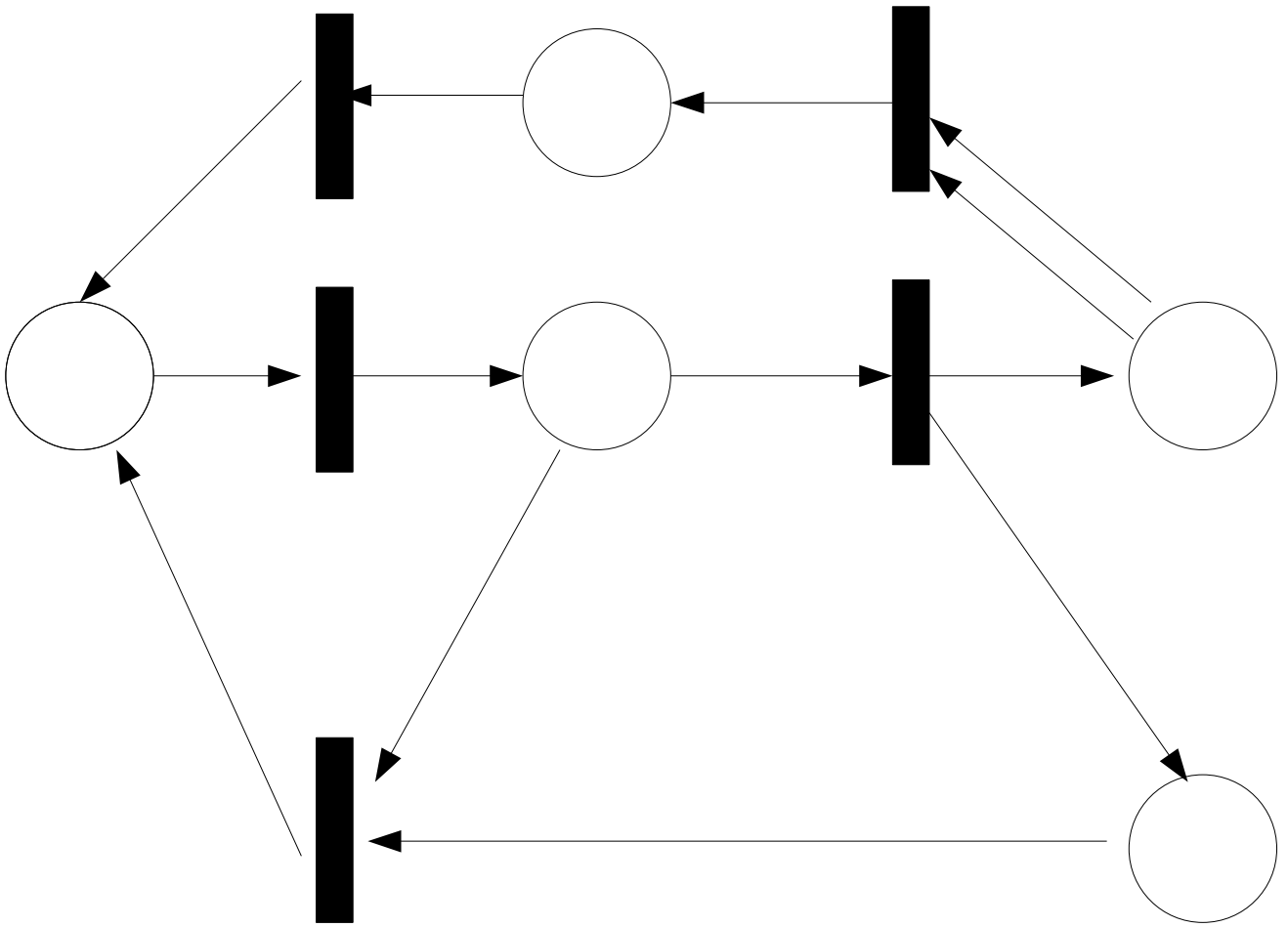
Bipartite graph

- Petri nets are bipartite graphs
- In bipartite graphs the set of vertices V can be divided into two disjoint subsets V_1 and V_2 such that any edge always connects vertices from different subsets
- The graph is sometimes denoted as $G:=(V_1+V_2, E)$

Petri Nets as graphs

- In Petri nets nodes of the first subset of vertices are called places, nodes of the second – transitions
- The symbol of a place is a circle or an ellipse
- The symbol of transition is a solid bar or a rectangle
- The edges of the graph are called arcs

Sample simple Petri net



Tokens

- In order to describe dynamics of Petri nets (and being able to “execute” them) another concept is introduced – that of a token
- The tokens are denoted by a solid dot and can be placed inside the place symbol
- They indicate presence or absence of, for example, resource
- Places can hold any number of tokens or only a limited number (capacitated places)

Transitions

- Tokens are used to describe enabling of transitions
- If an arc is drawn from a place to a transition, it indicates that a token in the place is required to enable the transition
- If many arcs are drawn (multigraph!), its number indicates the number of required tokens
- The transition is enabled iff for all arcs coming to the transition the condition of the required tokens are met

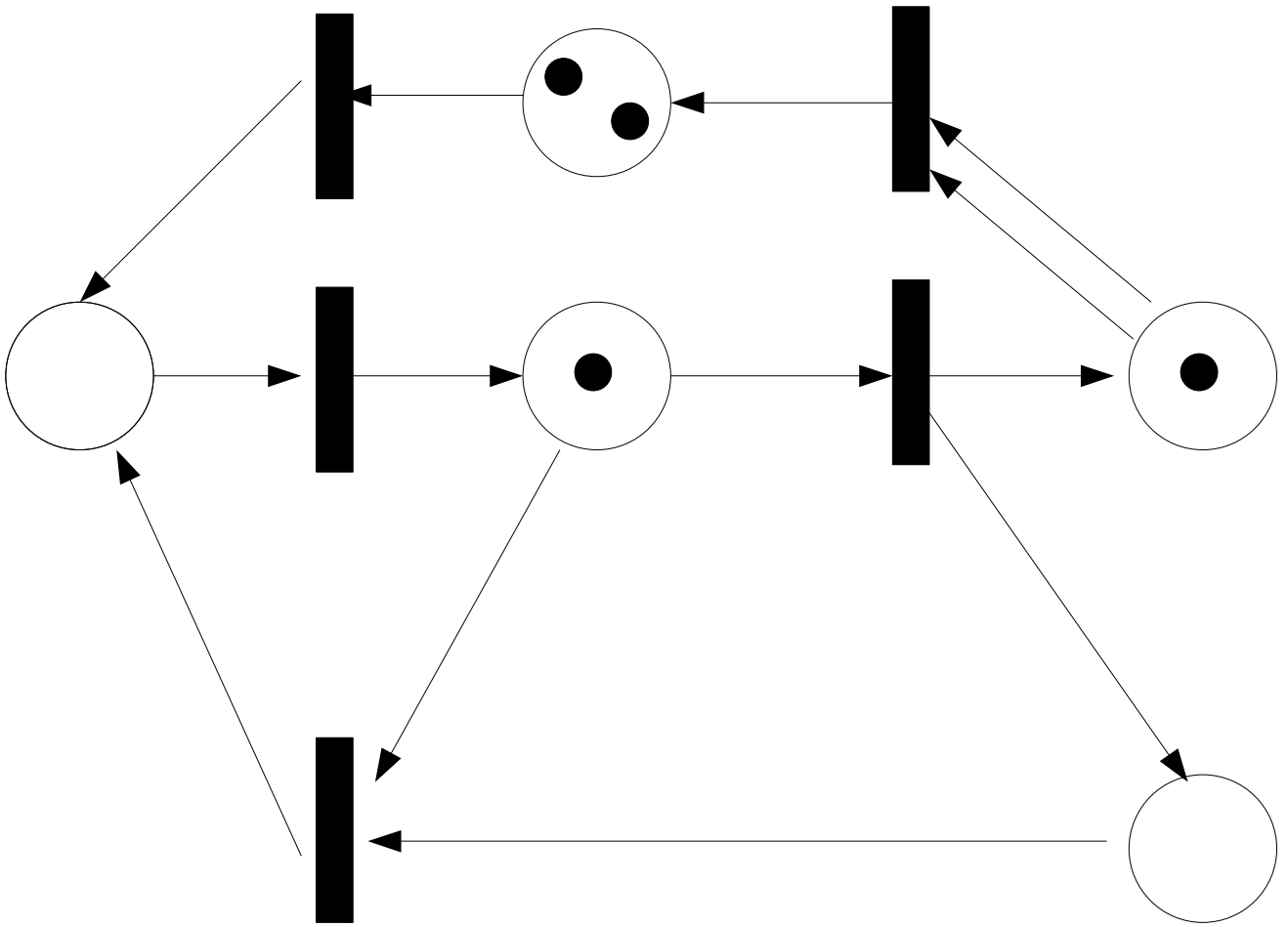
Input and output sets

- In order to clarify the description the concept of input and output sets is introduced
- The input set (preset) of a transition t, denoted ●t, is a set of all places for which there are arcs going from these places to the transition t
- The output set (postset) of a transition t, denoted t●, is a set of all places for which there are arcs going from transition t to these places
- Similar definitions apply to input and output sets of a place p, denoted by ●p and p●, respectively

Marking

- Marking of a Petri net is distribution of tokens in this net
- It is a mapping $P \rightarrow \{0, 1, 2, ...\}$ that describes the number of tokens present in each place
- The marking of the net at the beginning of an analysis is called initial marking

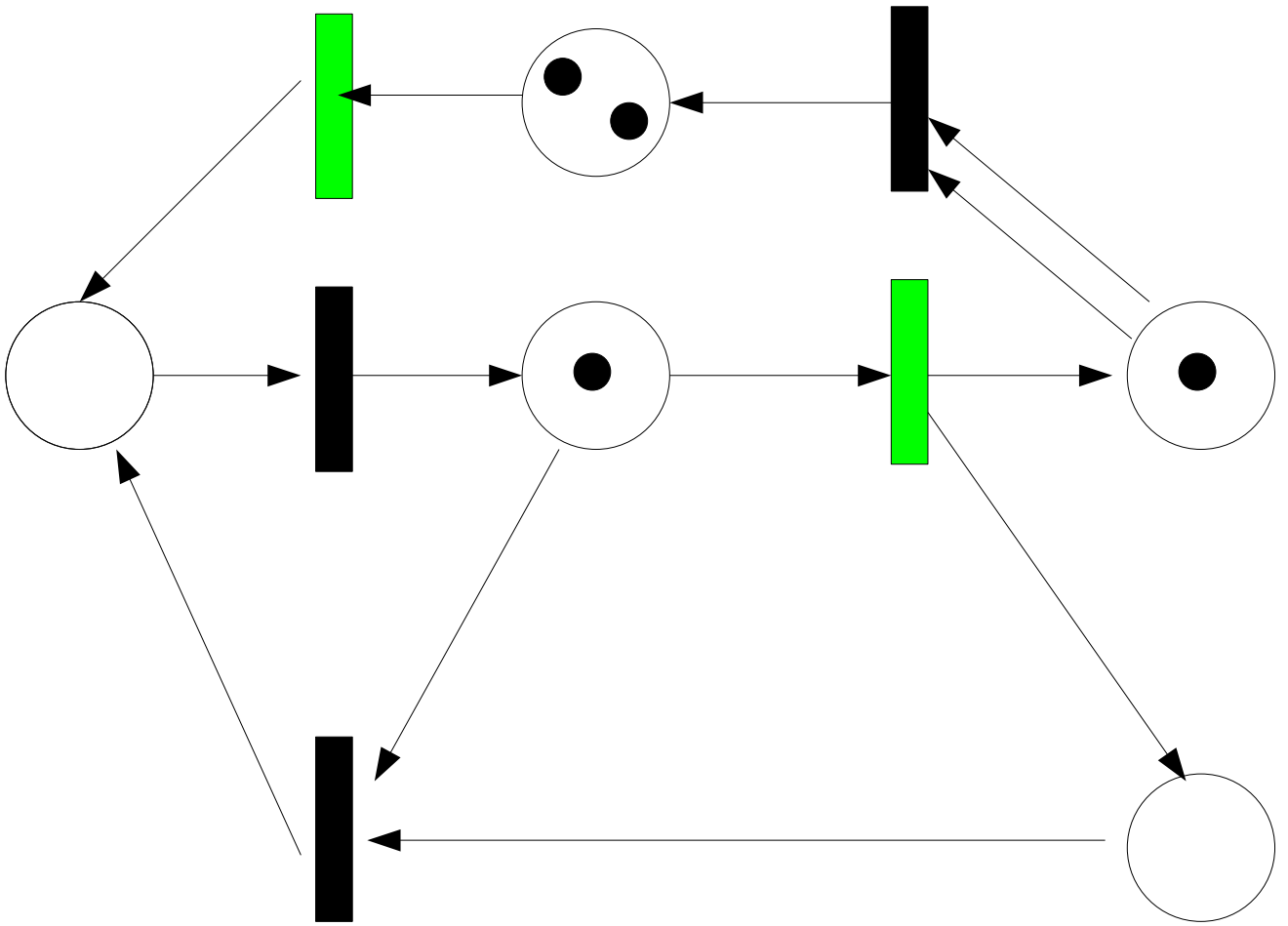
Sample marking



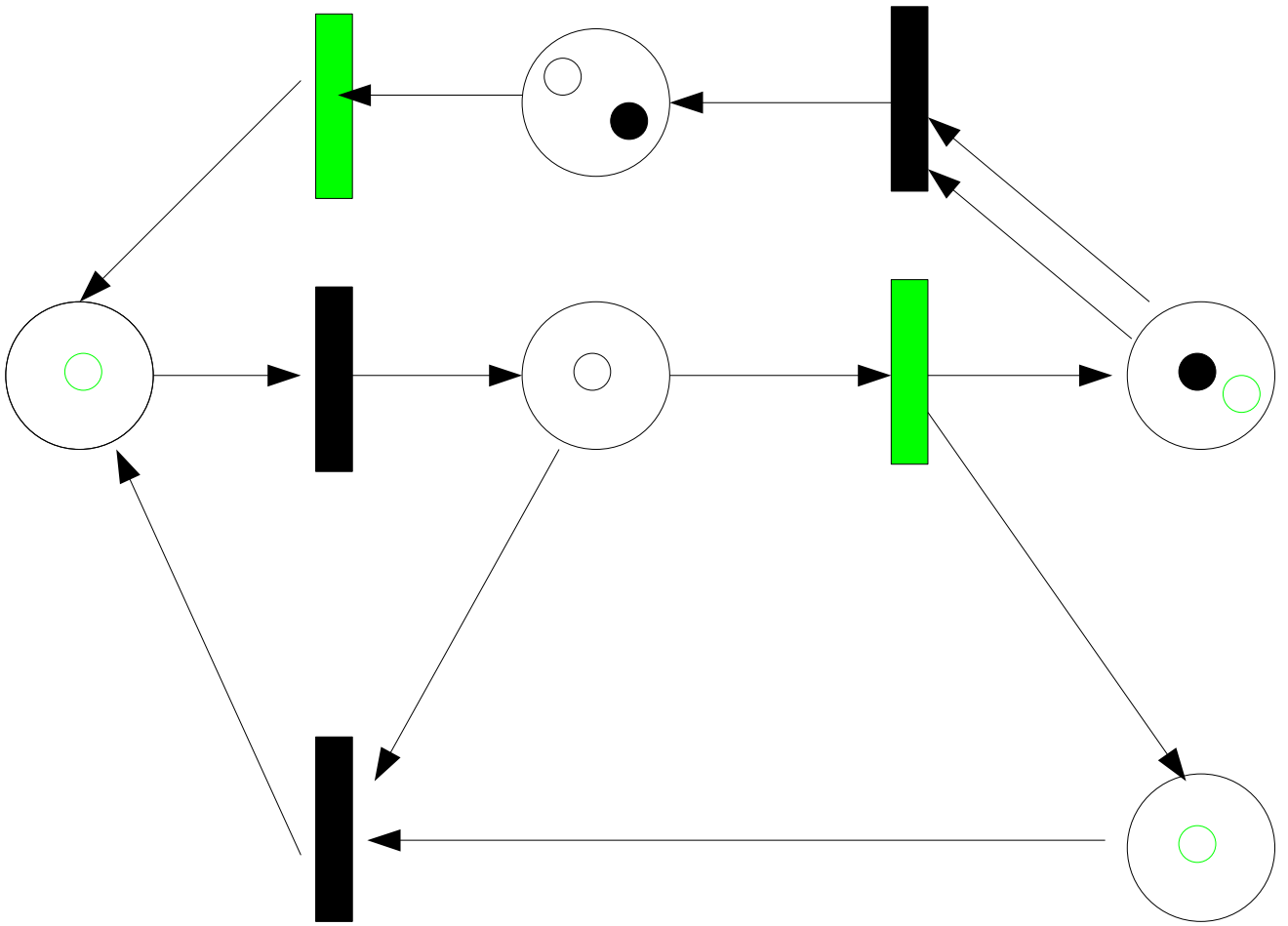
Firing

- If a transition is enabled, it can fire
- When a transition fires, tokens are removed from all its input places (taking into account multiple arcs!)
- After that, tokens are inserted into all its output places – again taking into account multiple arcs
- The number of the tokens removed and inserted may be different!
- Firing sequence is a sequence of transitions firing

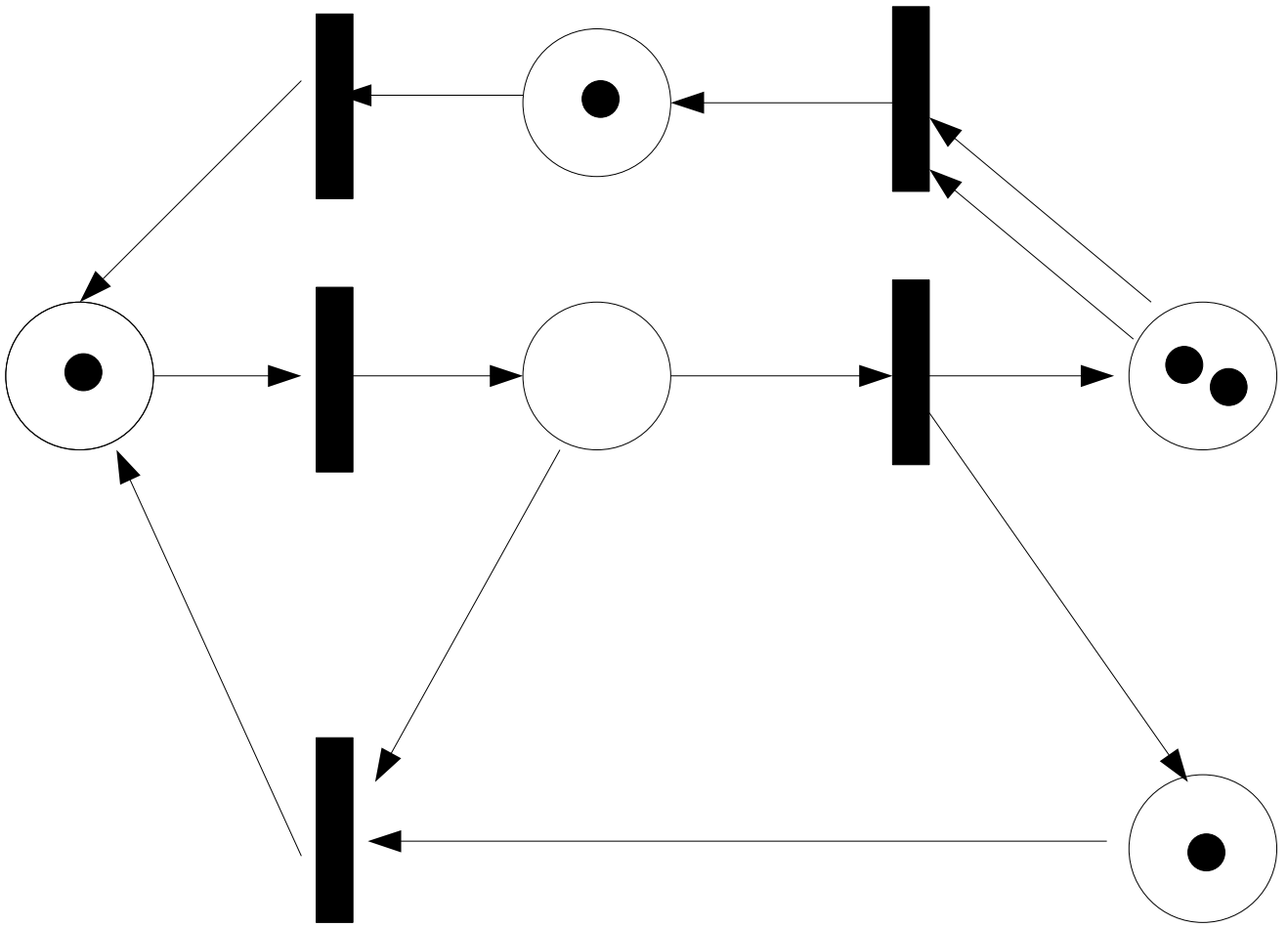
Firing



Firing



Firing



Usefulness of Petri nets

- Petri nets can be used to model complex processes
- Petri nets can be simulated (executed) in order to illustrate and test system behaviour, benchmark its speed etc.
- It is possible to perform a formal analysis of Petri net to find possible problems of the systems (for example deadlocks)
- For different applications the places and transitions may have different interpretations

Interpretations of places and transitions

Input places	**Trasitions**	**Output places**
required resources	task	freed resources
input data	computations	output data
input signals	signal processing	output signals
buffers/registers	processor	buffers/registers

Some sample simulations

In separate files...

Properties of Petri nets

- Petri nets properties are basis for their formal analysis
- The following properties are commonly used:
  - Reachability
  - Liveness
  - Boundedness

Reachability

- Reachability set $R(M_0)$ is a set of all possible markings reachable from the initial marking M_0
- If a marking M is in the reachability set $R(M_0)$, it means that there exists a firing sequence transforming M_0 into M
- This is an useful property for analysis of systems. We can ask whether:
  - A desired state can be reached at all?
  - Is it possible for a system to arrive at an undesired (erroneous) state?

Reachability and reachability graph

- Reachability can be analysed by building a reachability graph
- It is a directed graph where nodes represents markings and edges – transitions between two markings
- The graph is constructed by finding all possible transitions from the initial marking – this gives a set of markings reachable from the initial one, then all possible transitions from the previously discovered markings, and so on
- May lead to extremely large graphs

Liveness

- A marking M_x of network is live if, for any transition t and for every reachable marking M_y there exists a firing sequence from M_y that includes t
- In other words, every transition of the net can fire an infinite number of times
- A Petri Net PN is structurally live, if any initial marking of PN is live
- Liveness may be used to model the occurrences of deadlocks

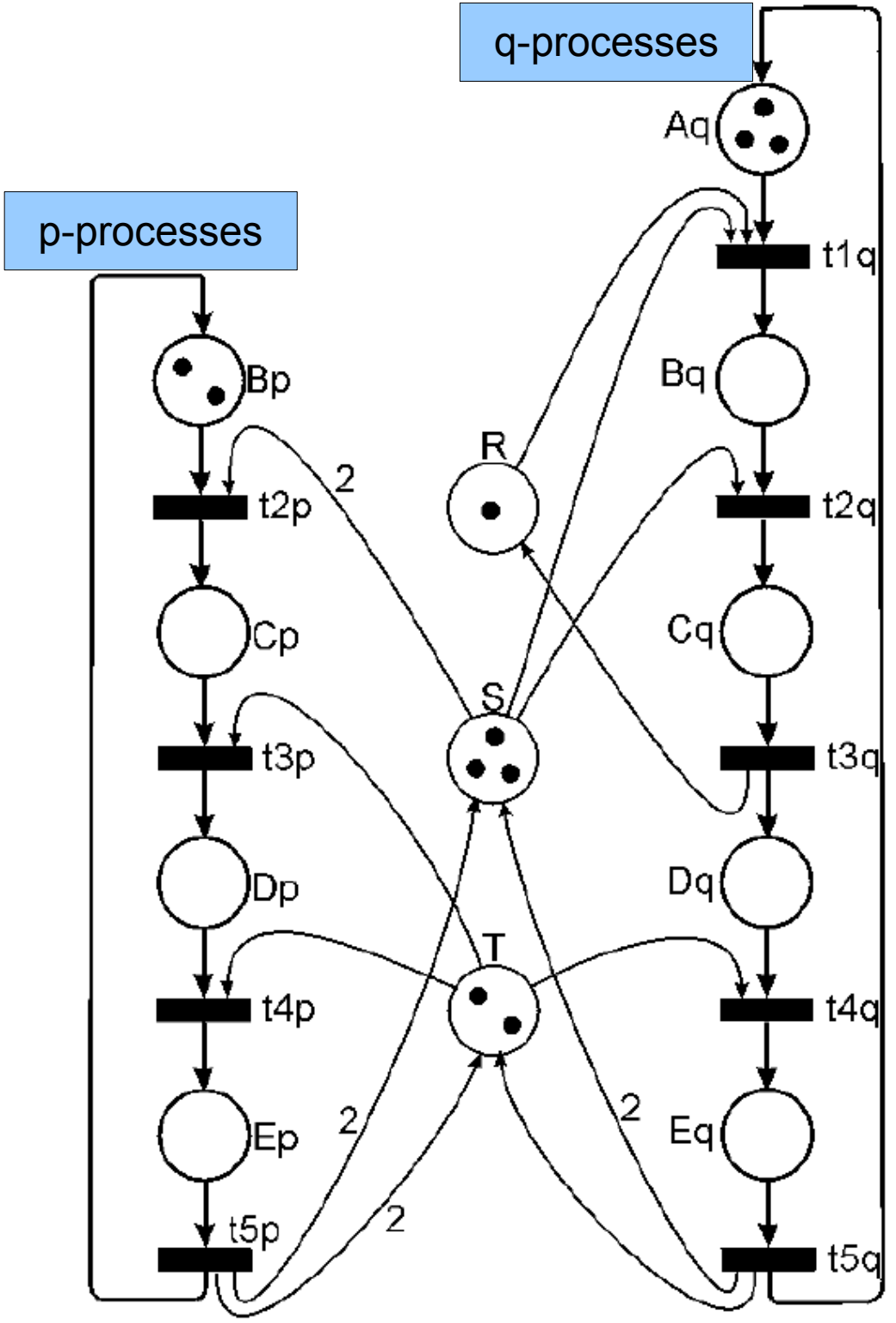
Boundedness

- A marking M_x is bounded if there exists a positive integer k such that for every reachable marking M_y - element of the reachability set $R(M_x)$ - the number of tokens in each place is bounded by k.
- A Petri net is structurally (inherently) bounded if all of its initial markings are bounded
- In other words, no reachable state can at any place contain more than k tokens
- If k equals one, the marking is said to be safe
- This property is useful for modelling limited (bounded) resources

Limitations of Petri nets

- The type of Petri nets described up to this point is called Place/Transition nets
- They have a number of limitations:
  - Inability to model similar (but not identical) processes using one net
  - All tokens are identical
  - No way to represent additional properties – there is no way to associate any additional data with token

Sample net


q-processes
p-processes
Aq
t1q
Bq
t2q
Cq
t3q
Dq
t4q
Eq
t5q
Bp
t2p
Cp
t3p
Dp
t4p
Ep
t5p
R
S
T
2
2
2
2

More complex Petri nets

- In order to overcome these problems, a number of solutions extending the initial approach has been proposed:
  - Coloured Petri Nets
  - Hierarchy Petri Nets
  - Object Petri Nets
  - Prioritised Petri Nets
  - Timed Petri Nets
    - Stochastic Petri Nets

Coloured Petri Nets (CPNs)

- Allow to construct more compact models by sharing similar parts between various processes
- Handling additional data is possible
- This is mainly achieved by transition from identical tokens that represent binary value to typed tokens that can hold any kind of data

Coloured Petri Nets

- The data value attached to a token is called token **colour**
- The data value can be a simple number, a string, a structure consisting of some fields etc.
- The type of the value is called **colour set** – it is identical to type in programming languages
- Each place in CPN has its colour set (type)

Colour sets

- Before any coloured tokens are created, colur sets (types) must be defined
- Types may be defined in an arbitrary way, but in order to use computer tools, a formal method is required
- One of such methods is CPN ML language

Marking and initial marking

- Initial marking in CPNs is determined by evaluation of initialization expressions that are attached to places (by convention an underlined expression next to a place)
- As the tokens may now carry complex information, their representation by presence or absence of dots is not useful
- Instead, the number of tokens is denoted by a number in a circle next to the place, and the token values are listed below this circle

Arc expressions

- As the tokens are more capable, transitions should have a way to use the information carried by the tokens
- This is possible thanks to arc expressions
- Arc expressions evaluate to a set of tokens that determine type and number of tokens “passed” through arc
- Arc expressions can contain a number of variables, operations on these variables (e.g. comparison), logical operators etc. Therefore they may evaluate to different values for different tokens

Arc expressions

- Arc expressions can be therefore treated as functions:
  - Tokens (their types and values) are arguments to the expressions
  - Tokens (their types and values) are return values from the expressions and determine token flow

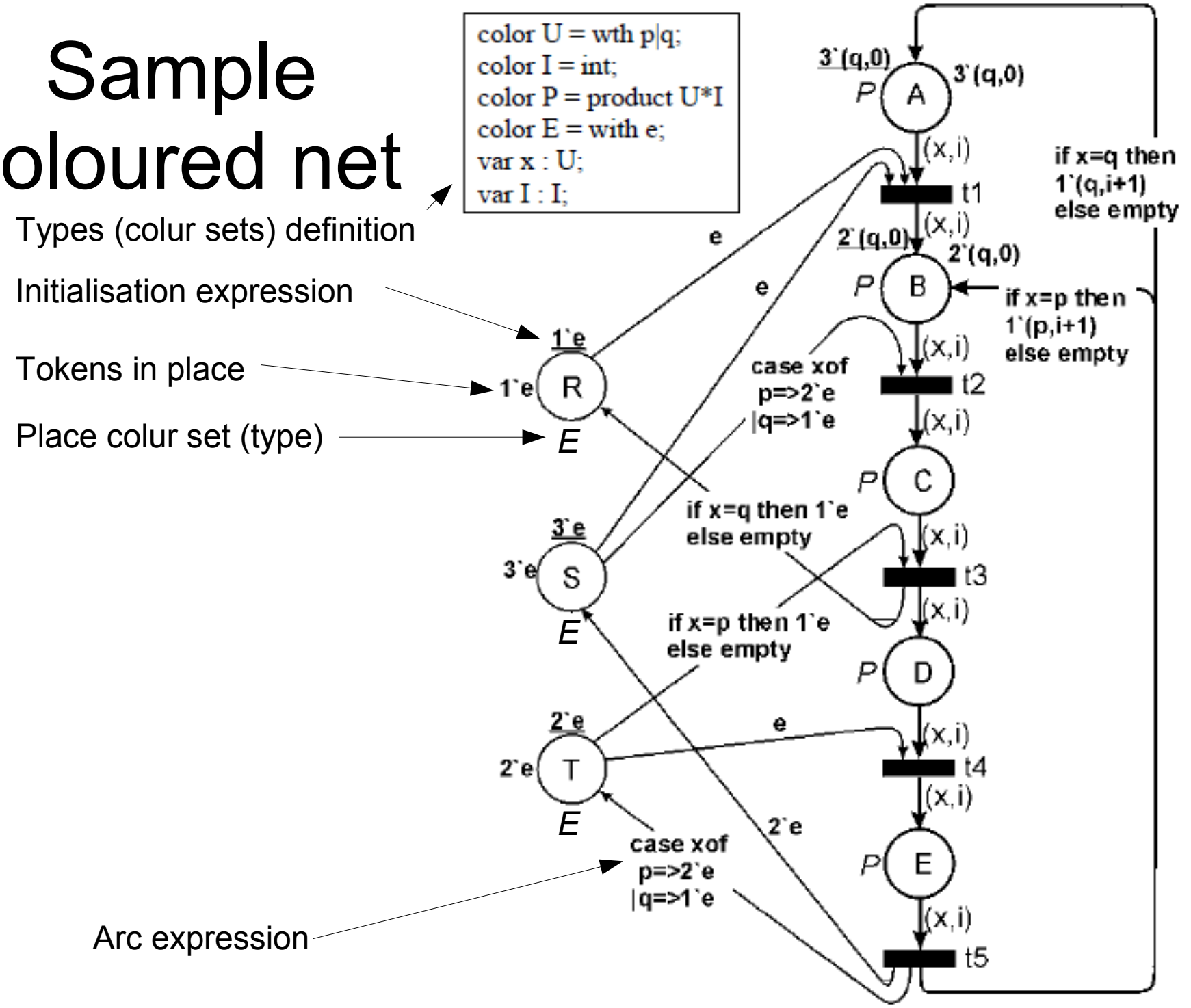
Arc expressions and enabled transitions

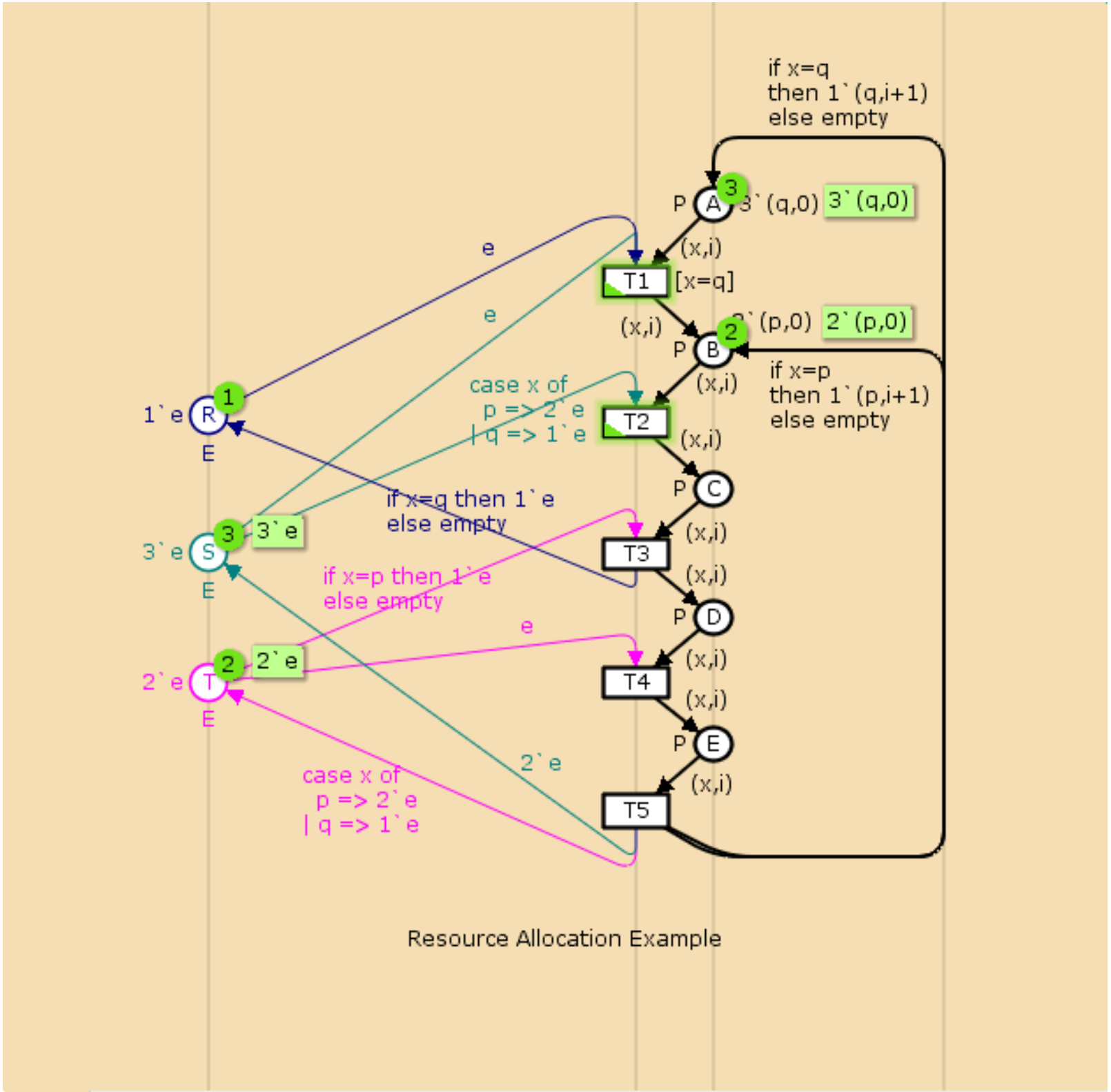
- Arc expression going to transition nodes determine if the transition is enabled
- In CPNs, thanks to arc expressions, enabling of a transition is conditioned not only on the presence of tokens in the input places, but also on their values

Guards

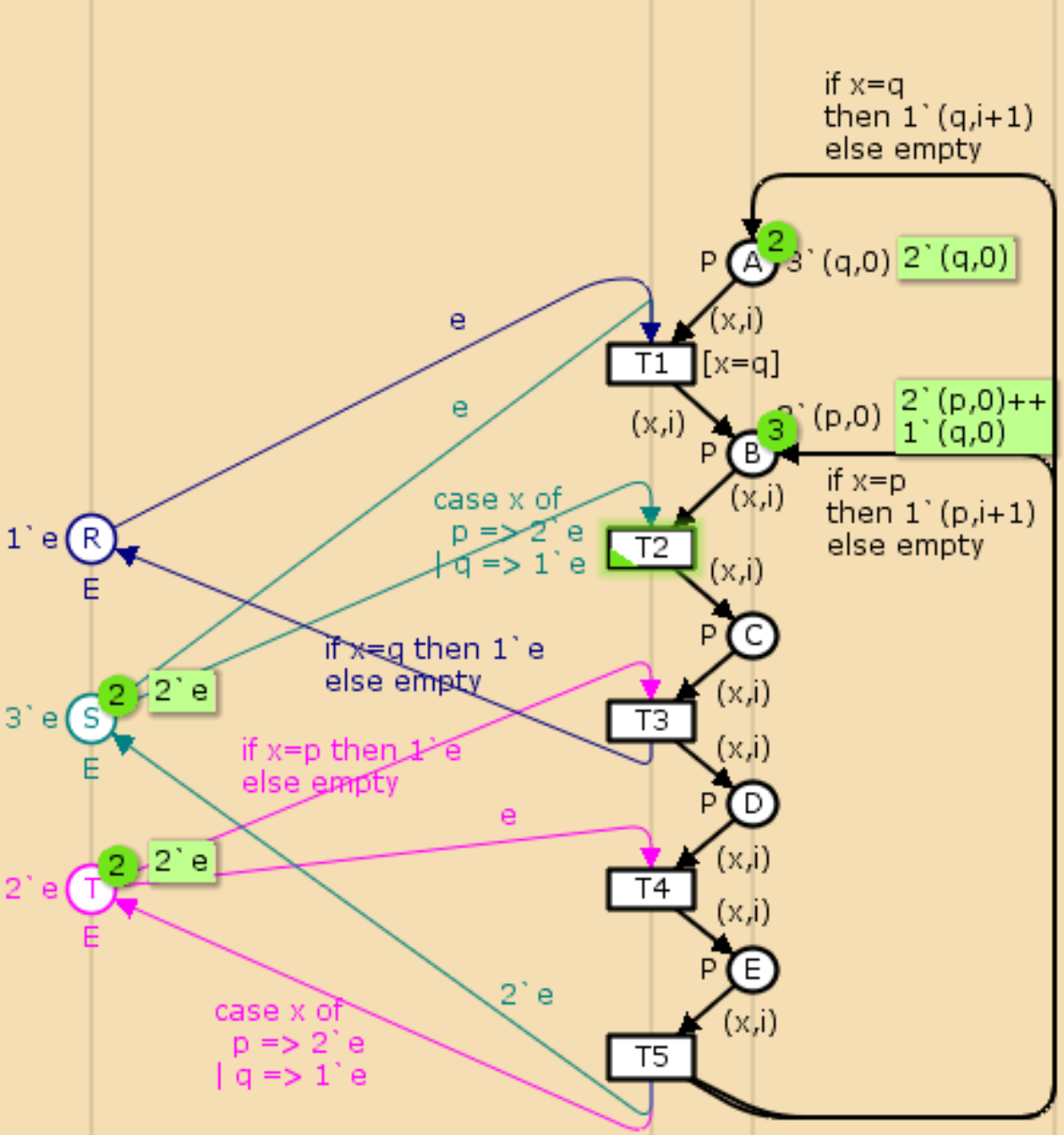
- Guard is a boolean expression applied to a transition node
- By convention it is placed in square parentheses next to a transition
- Guard is an additional condition that has to be fulfilled in order to enable transition

Sample coloured net





Resource Allocation Example



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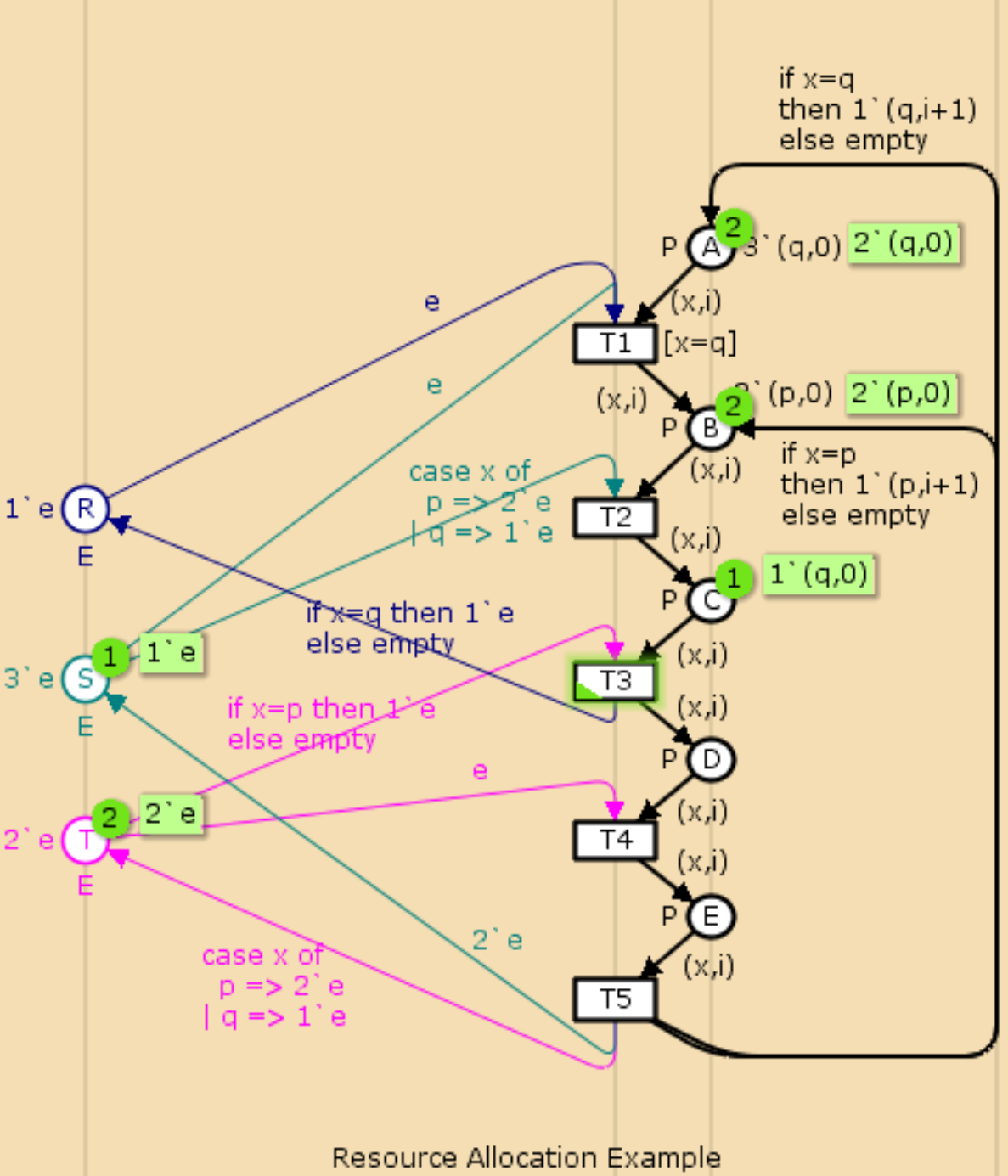

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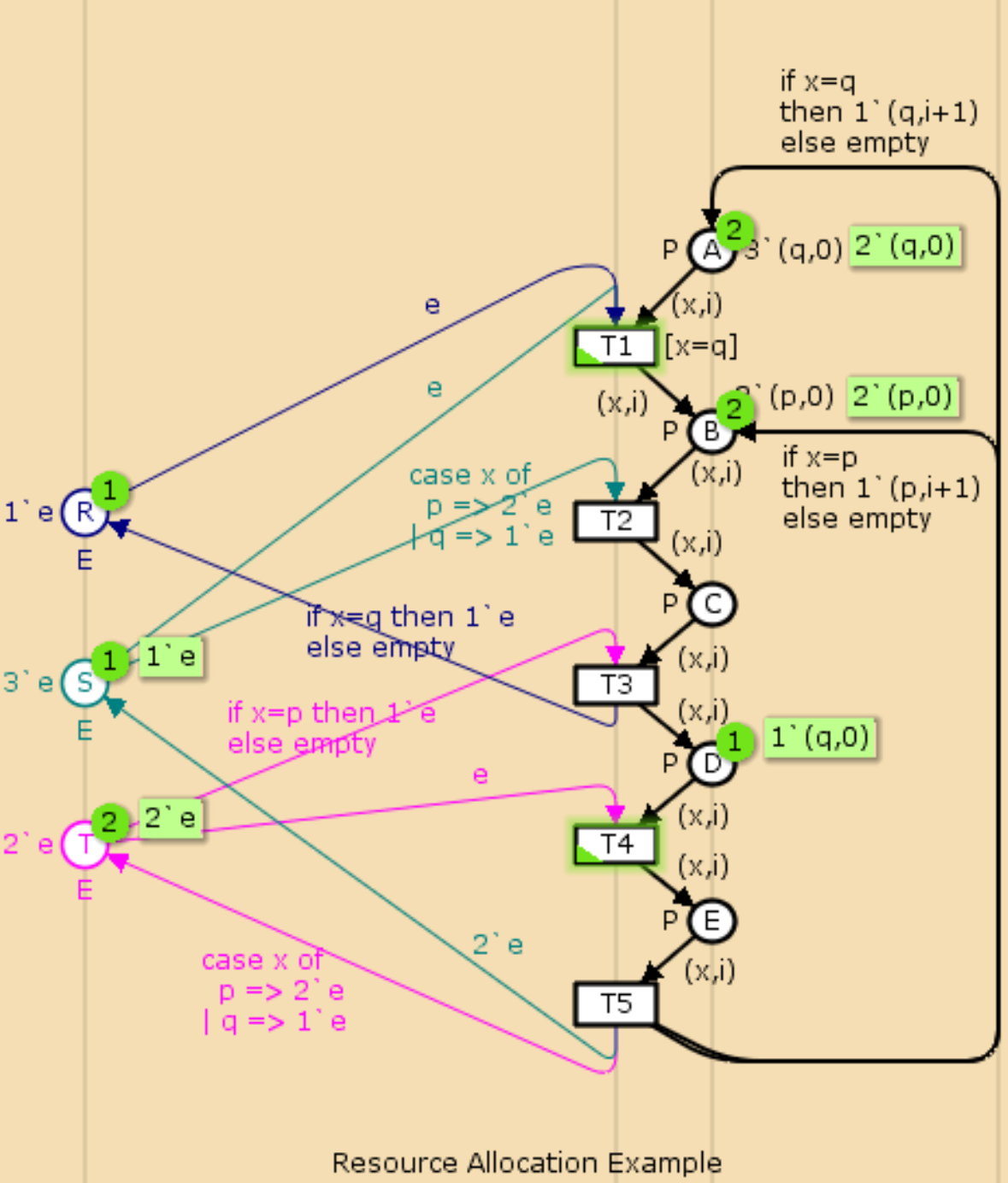

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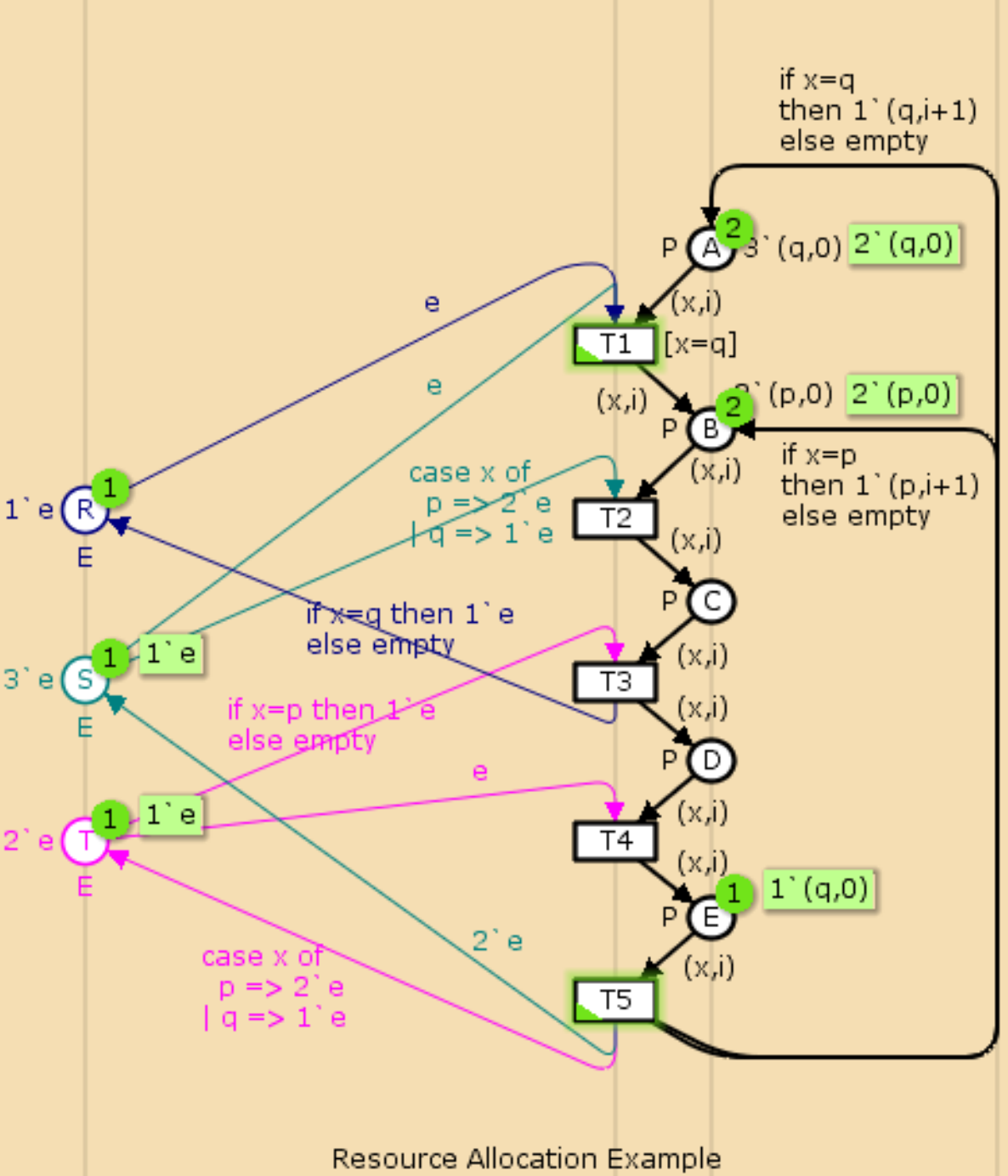

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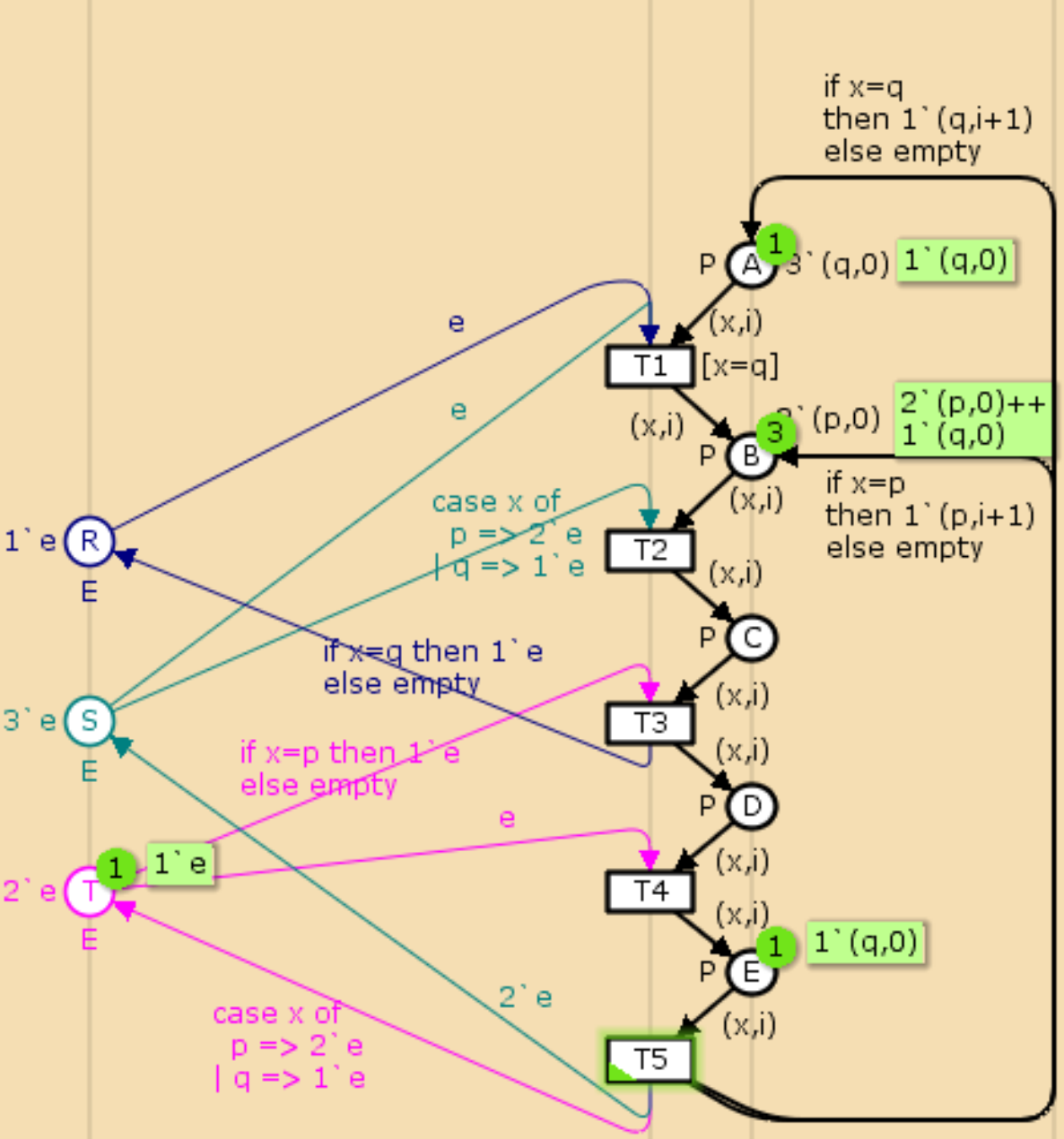

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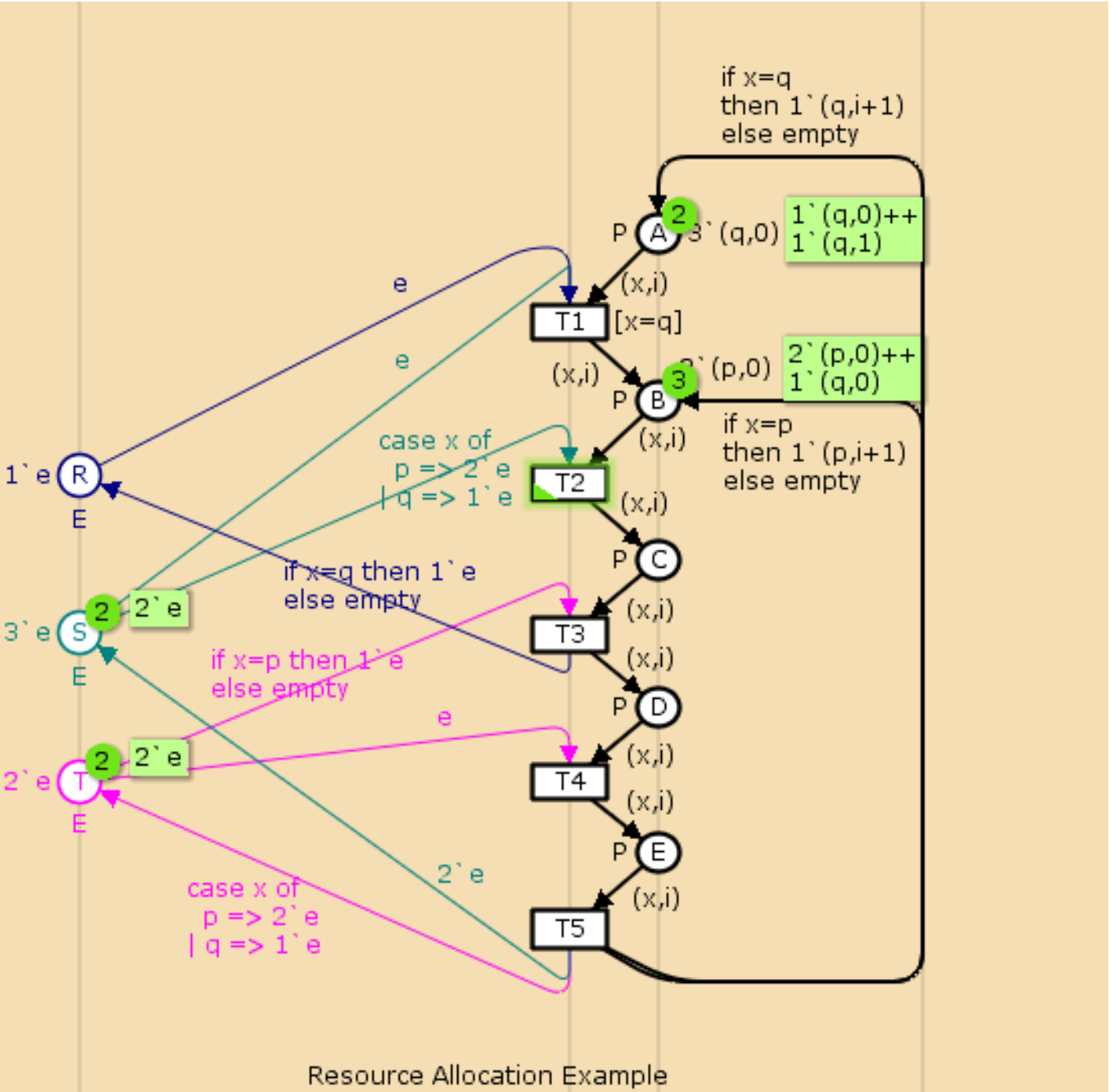

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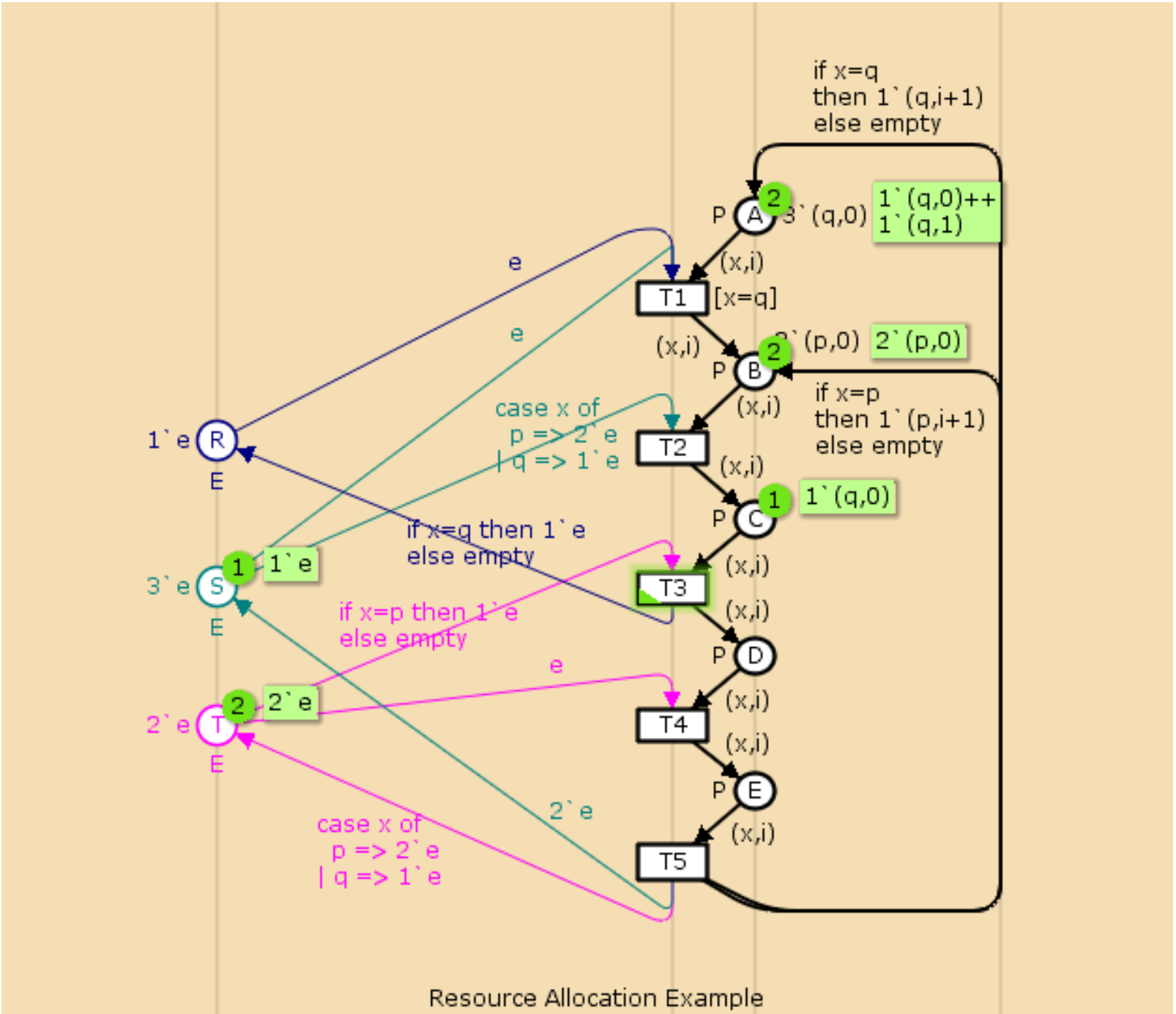

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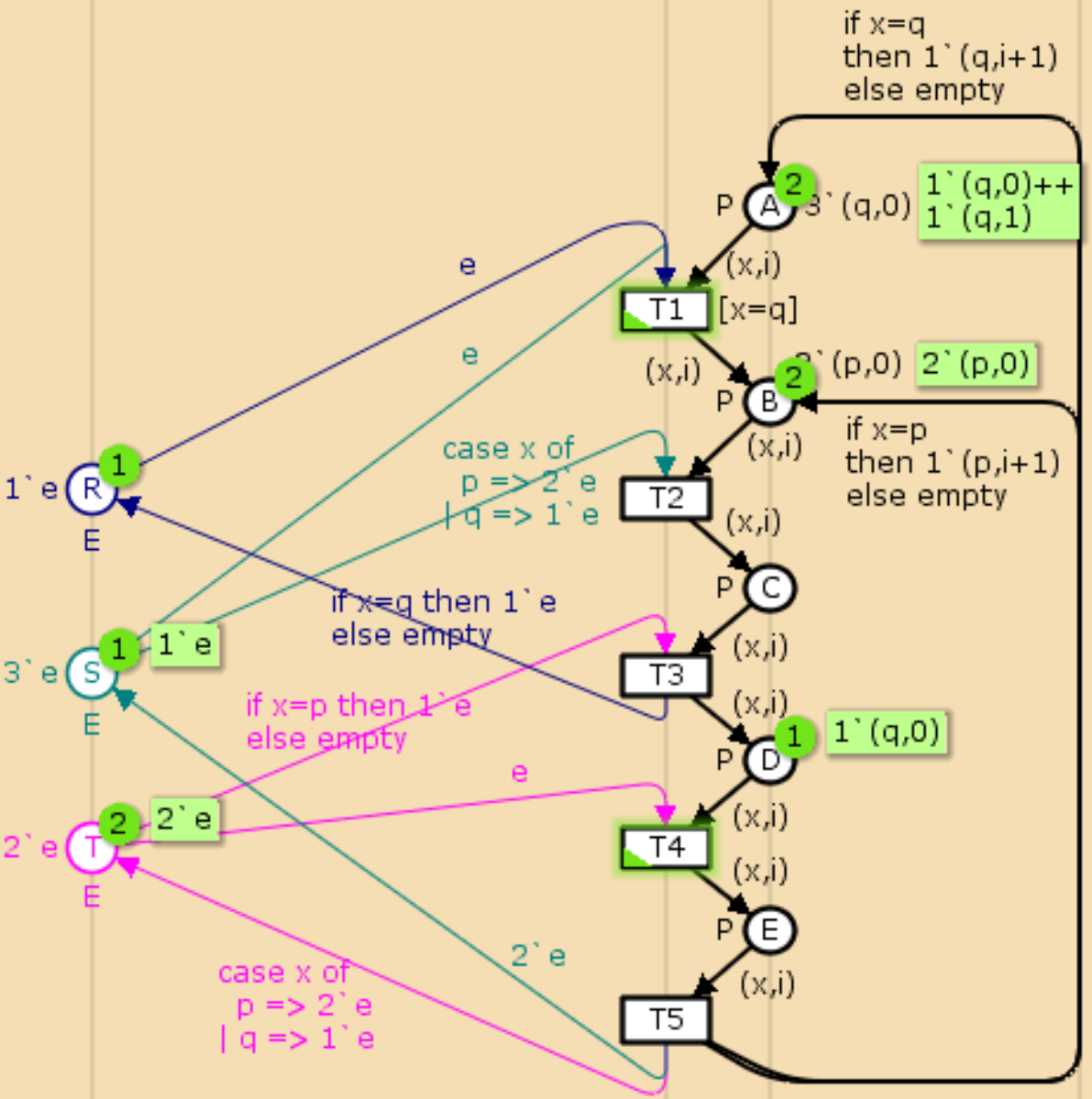

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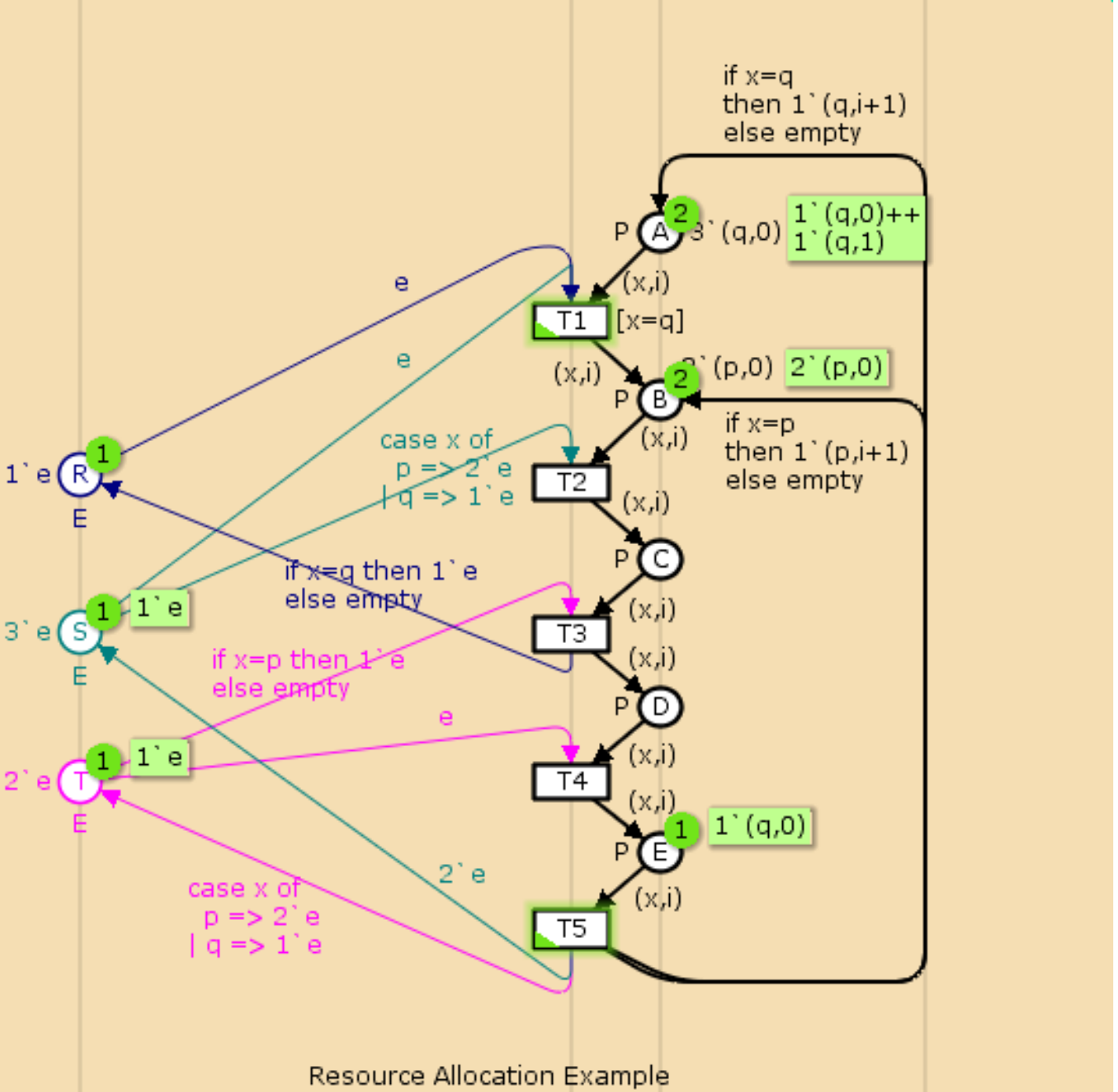

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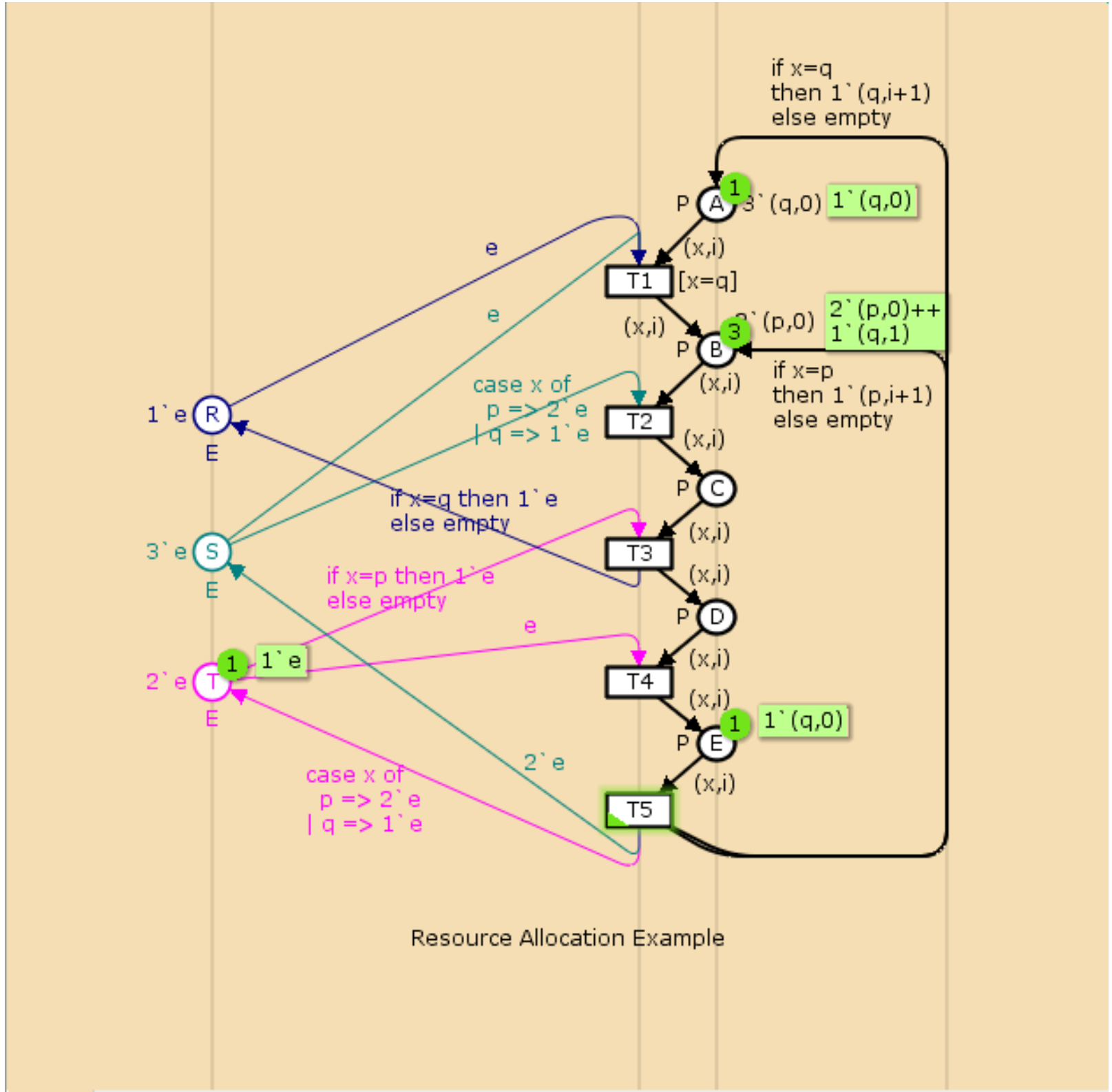

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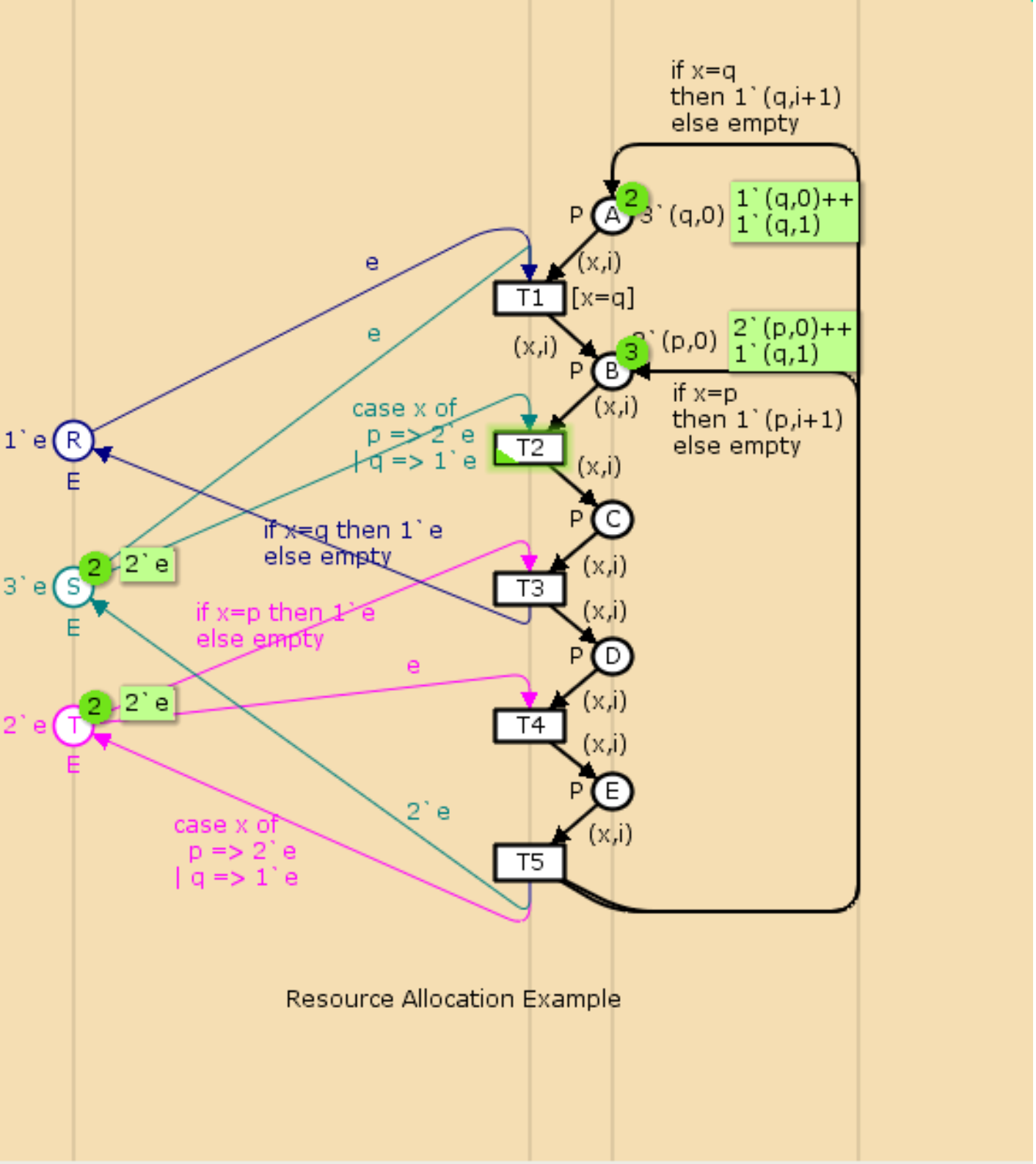
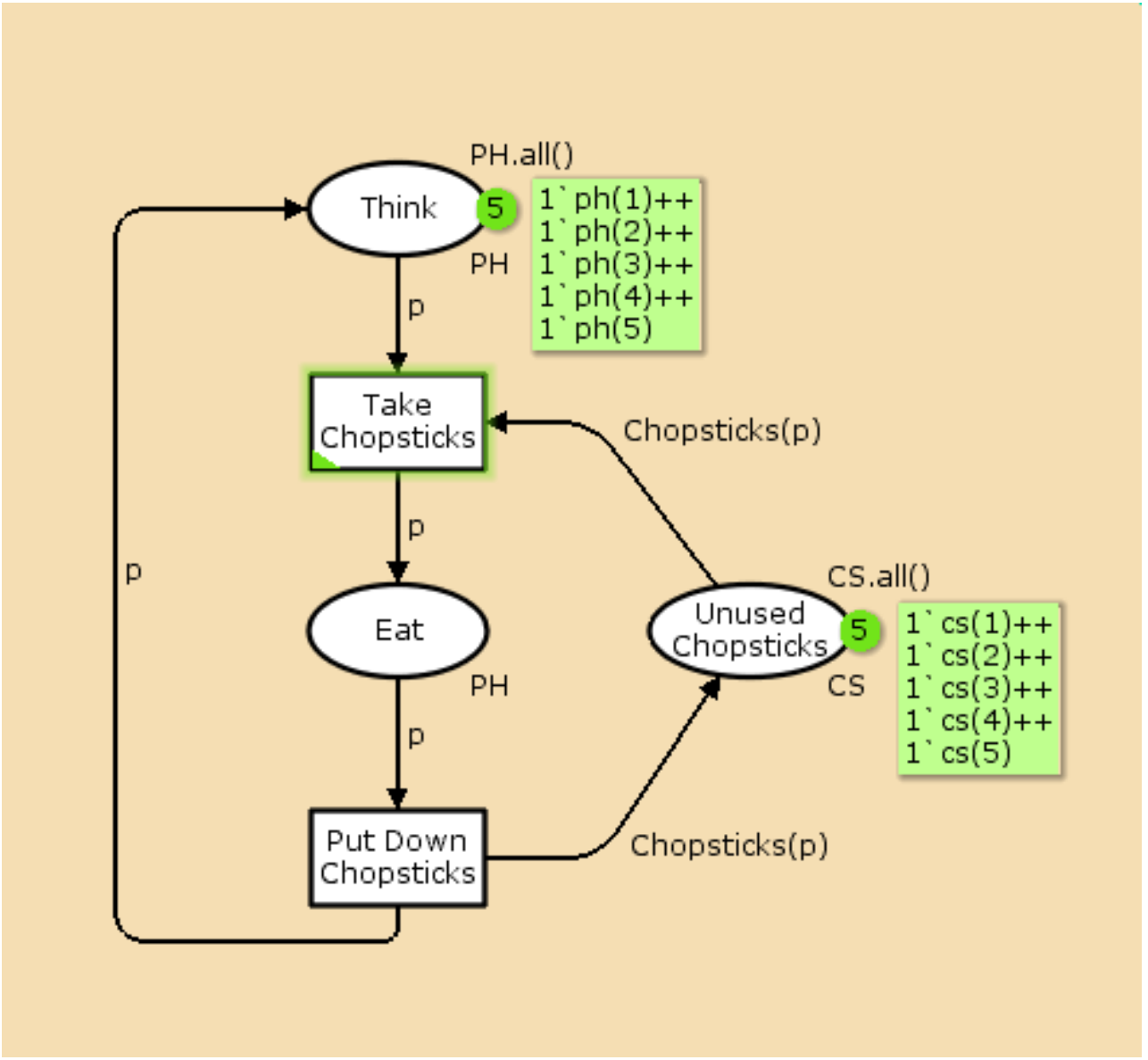


Resource Allocation Example

Coloured version of dining philosophers



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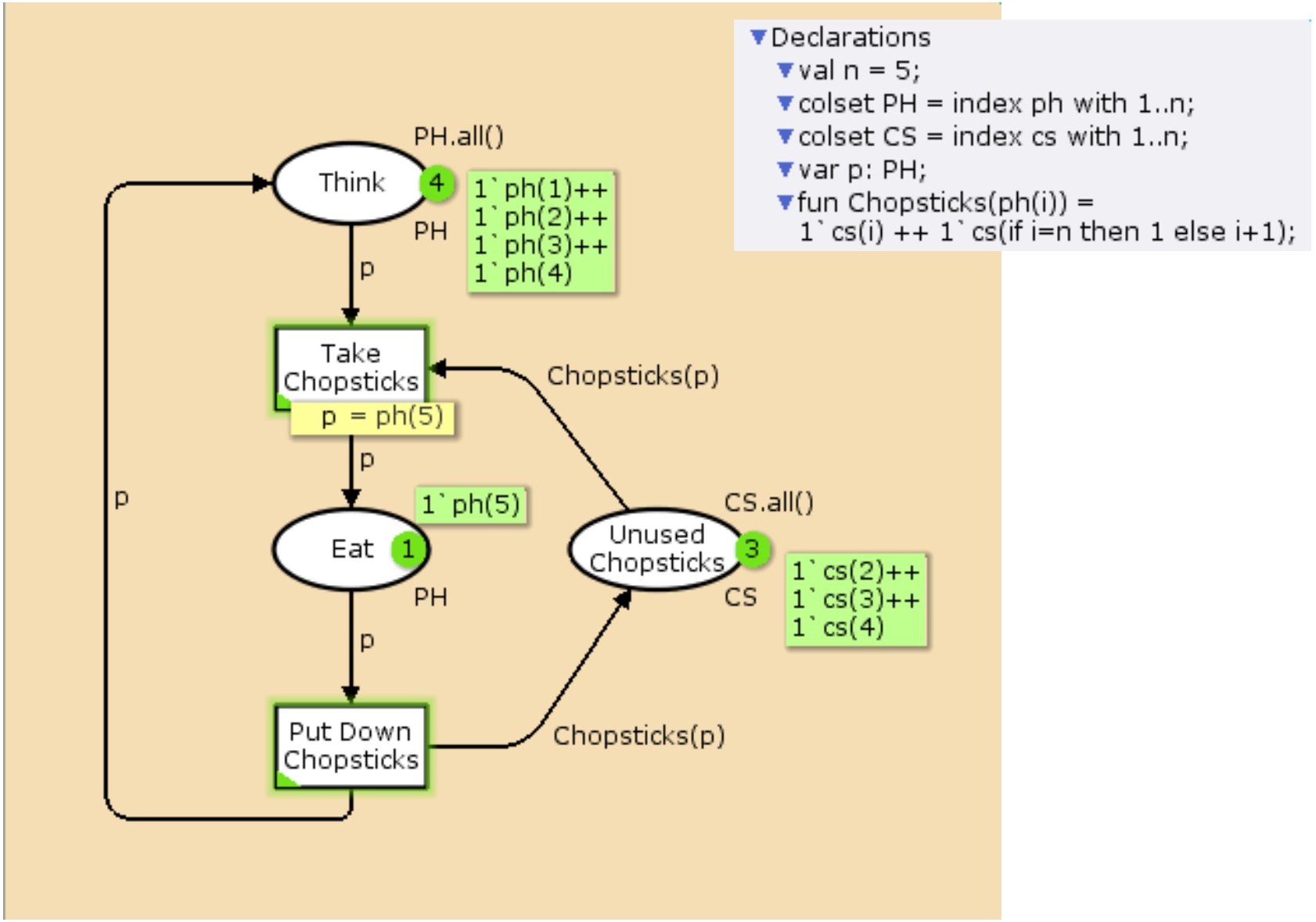

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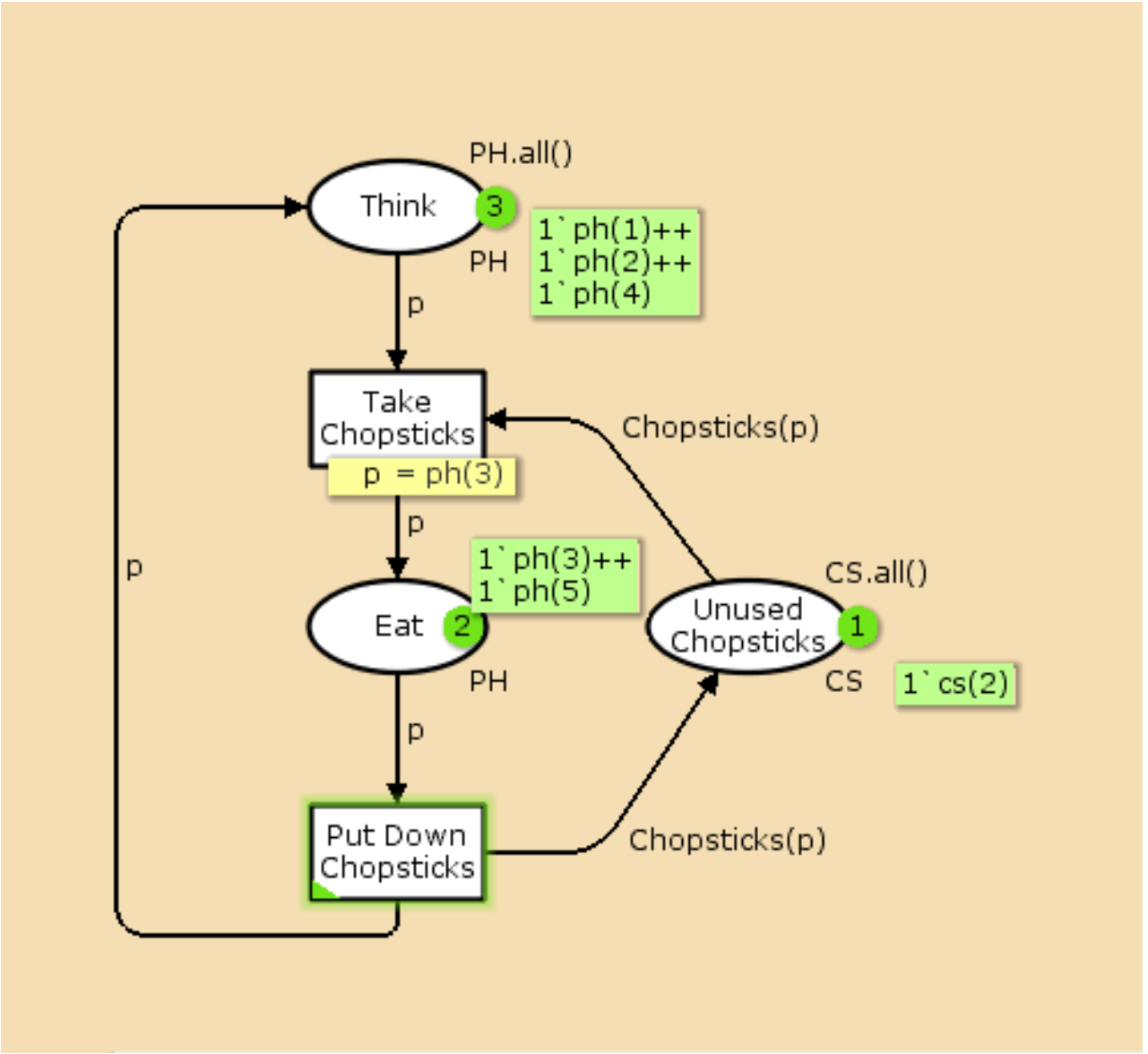

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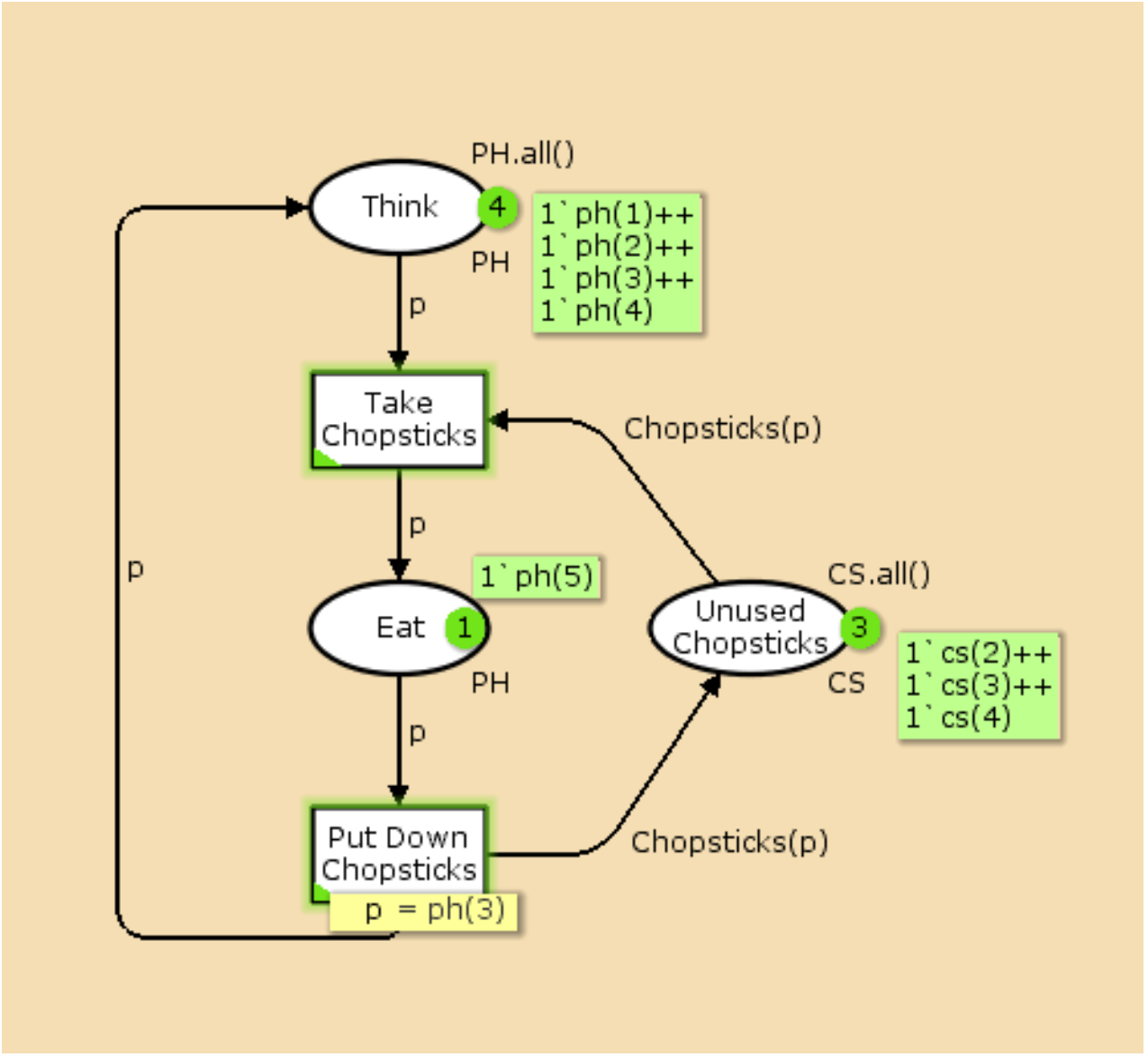


Software for Petri nets – CPN Tools

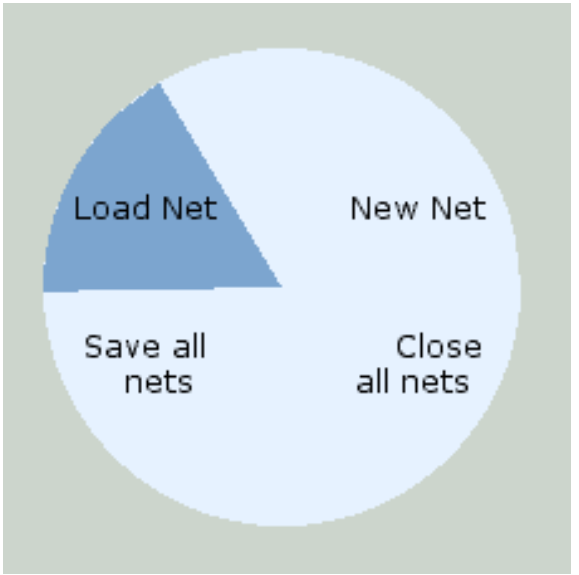
- http://wiki.daimi.au.dk/cpntools/_home.wiki
- Now (May 2007) in version 2.2.0
- Created by CPN Group, University of Aarhus, Denmark
- Runs on Win2K, Win XP, Linux Fedora Core 2, probably other Linux distributions
- Requires hardware OpenGL acceleration, may not work with older graphics cards

CPN Tools interface

- CPN Tools uses a clever and intuitive UI, however, it is quite different from typical GUI – this may pose problems during first contact
- The screen is divided into “Index” (left side) and “Workspace” (the rest)
- Although the Index can be used for some operations, typical user interaction is different

CPN Tools interface

- Many functions are accessible by clicking and holding right mouse button – this brings up a circular context menu
- Some items can be dragged from the Index into Workspace – they expand into toolboxes



CPN Tools interface