Electronic Technology Design and Workshop

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DMCS room 2

2007

Electronic Technology Design and Workshop

Lecture 3

Electronic circuit analysis

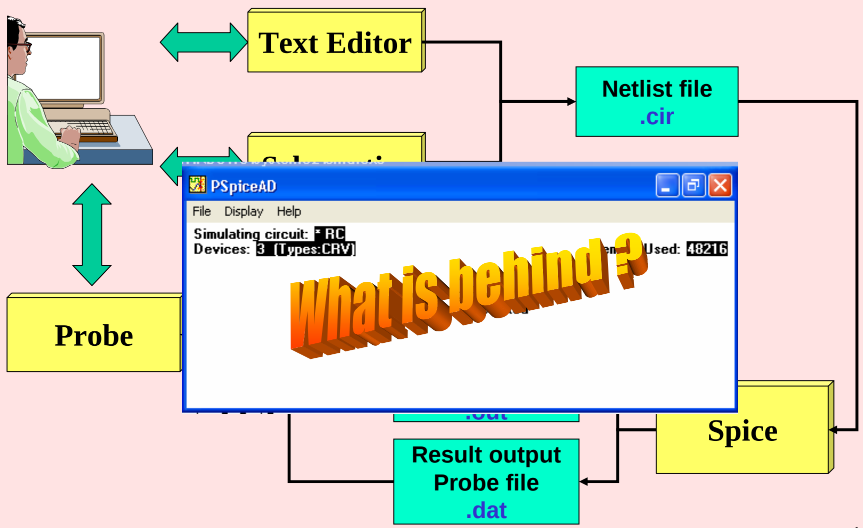
The SPICE – behind the GUI

ETDW course road map

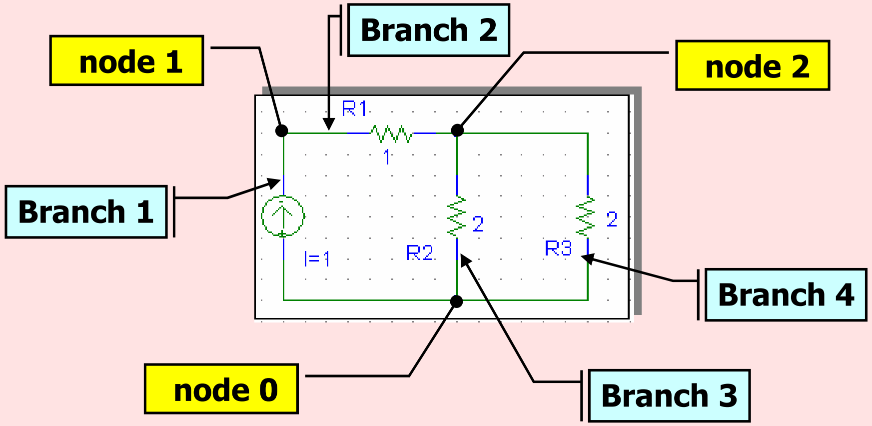
- ✓ Schematic edition, libraries of elements
- ✓ **Circuit simulation & netlist generation**
- ✓ Microelectronics - full custom design and simulation
- ✓ Microelectronics - simple layout synthesis
- ✓ Hardware description languages - behavioural description
- ✓ Logic & sequential synthesis - programmable logic devices
- ✓ PCB design – auto-routing

Project - bringing the pieces together

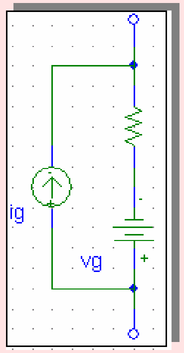
MicroSim - Data Exchange Organization



Circuit scheme



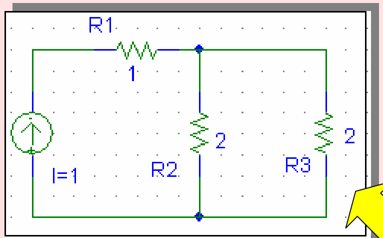
Fundamental branch



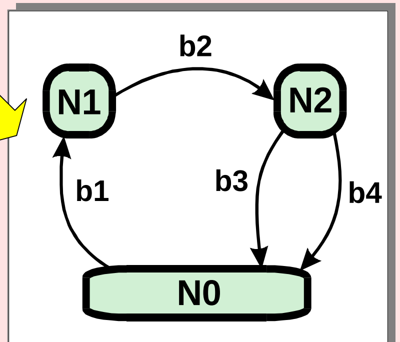
Fundamental branch consist of:

- Voltage source
- Current source
- Impedance

Network Graph – circuit topology



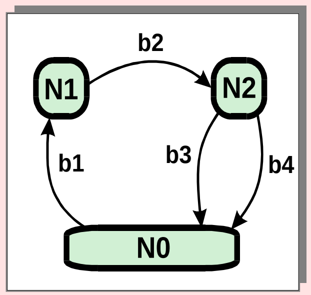
Sample Circuit



Network Graph

Notice that all branches are **oriented** !

Circuit topology



- Nodes: N0, N1, N2
- Branches: b1, b2, b3, b4
- Nodal voltages: V_{N1}, V_{N2}
- Branch Voltages: V_{b1}, V_{b2} , V_{b3}, V_{b4}
- Branch Currents I_{b1}, I_{b2}, I_{b3}, I_{b4}

Vector representation

$$\mathbf{V}_b = \begin{bmatrix} V_{b1} \\ V_{b2} \\ \vdots \\ V_{bM} \end{bmatrix} \quad \mathbf{I}_b = \begin{bmatrix} I_{b1} \\ I_{b2} \\ \vdots \\ I_{bM} \end{bmatrix} \quad \mathbf{V}_n = \begin{bmatrix} V_{n1} \\ V_{n2} \\ \vdots \\ \vdots \\ V_{nN} \end{bmatrix} = ?$$

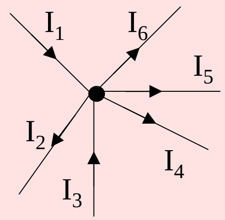
Branch Voltages **Branch Currents** **Nodal Voltages**

We are looking for nodal voltages ?

Kirchhoff's Current Law

For each node

$$\sum_{i=1}^{z} I_i = 0$$



For one sample node we can write

$$I_1 - I_2 + I_3 - I_4 - I_5 - I_6 = 0$$

How to generalize for all nodes?

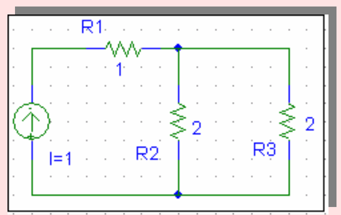
Incidence matrix

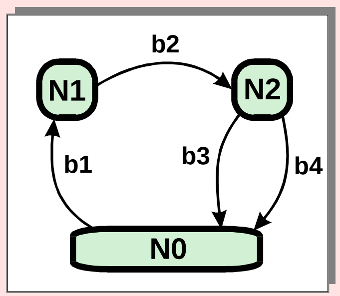
To apply **Kirchhoff's Current Law** to whole circuit, let's create the term:

INCIDENCE MARIX - **A**

$$a_{kj} = \begin{cases} 1 & \text{if current } i_j \text{ leaves node } k \\ -1 & \text{if current } i_j \text{ enters node } k \\ 0 & \text{if } i_j \text{ neither enters nor leaves node } k \end{cases}$$

Extraction from .CIR file



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Circuit Netlist

* Sample circuit

I1 1 0 1
R1 1 2 1
R2 2 0 2
R3 2 0 2

.DC
.END
```

Incidence matrix generating

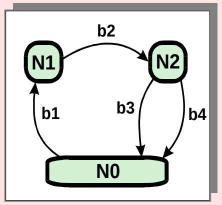
```
Circuit Netlist

* Sample circuit

I1 1 0 1
R1 1 2 1
R2 2 0 2
R3 2 0 2

.DC
.END
```

$$\mathbf{A} = \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} I1 & R1 & R2 & R3 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ N0 \\ N1 \\ N2 \end{array}$$



Incidence matrix vs reduced incidence matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

Can we remove one row without loosing the information ?

$$\mathbf{A}_{reduced} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

Kirchhoff's Current Law

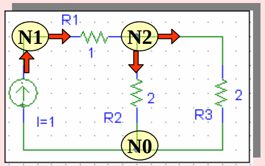
$$\mathbf{A}\ \mathbf{I_b} = \mathbf{0}$$

Matrix representation

$$\mathbf{A}_r\mathbf{I} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_{I1} \\ I_{R1} \\ I_{R2} \\ I_{R3} \end{bmatrix} = 0$$

Equation set representation

$$\begin{cases} -I_{I1} + I_{R1} = 0 \\ -I_{R1} + I_{R2} + I_{R3} = 0 \end{cases}$$



The Ohm's law

To get relationship between branch voltages v_b and branch currents i_b, let's use Ohm's Law:

$$\mathbf{v_b = r_b\ i_b}$$

or

$$\mathbf{i_b = y_b\ v_b}$$

For the whole circuit we need to move to matrix representation

$$\mathbf{V_b = R_b\ I_b} \qquad \mathbf{I_b = Y_b\ V_b}$$

R_b – branch resistance (impedance) matrix
Y_b – branch admittance matrix

Admittance matrix and sources vector

Circuit Netlist

```
* Sample circuit

I1 1 0 1
R1 1 2 1
R2 2 0 2
R3 2 0 2

.DC
.END
```

$$\mathbf{Y_b} = \begin{bmatrix} y_{I1}=0 & 0 & 0 & 0 \\ 0 & y_{R1}=1 & 0 & 0 \\ 0 & 0 & y_{R2}=1/2 & 0 \\ 0 & 0 & 0 & y_{R3}=1/2 \end{bmatrix}$$

$$\mathbf{I_g} = \begin{bmatrix} -1A \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

current sources vector

voltage sources vector

$$V_g = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Branch current with independent current source

But independent current and volatge sources also must be considered, so:

$$\mathbf{I_b = I^*_b - (I_g - Y_b V_g)}$$

and because there's no voltage on current sources, we have:

$$\mathbf{I^*_b = Y_b\ V_b}$$

and finally:

$$\mathbf{I_b = Y_b\ V_b - (I_g - Y_b V_g)}$$

Some maths

$$\mathbf{A\ I_b = 0}$$

$$\mathbf{I_b = Y_b\ V_b - I_g + Y_b V_g}$$

$$\mathbf{A\ (Y_b\ V_b - I_g + Y_b V_g) = 0}$$

$$\mathbf{A\ Y_b\ V_b - A\ I_g + A\ Y_b V_g = 0}$$

$$\mathbf{A Y_b\ V_b = A\ (I_g - Y_b V_g)}$$

… but the number of branches might be huge in comparison to number of nodes ...

Nodal voltage vs branch voltage

If there's taken to consideration that number of nodes is always less or equal than number of branches it is convenient to use Nodal-Voltage Vector $\mathbf{V_n}$ in place of Branch-Voltage Vector $\mathbf{V_b}$

Relation between $\mathbf{V_n}$ and $\mathbf{V_b}$ is following:

$$\mathbf{V_b = A^T\ V_n}$$

so finally we get

Fundamental relation for all linear networks

$$V_b = A^T V_n \qquad AY_b V_b = A(I_g - Y_b V_g)$$

$$AY_b A^T V_n = A(I_g - Y_b V_g)$$

what might be rewritten to

$$Y_n \quad V_n \quad = \quad I_n$$

where

Y_n - Node-Admittance Matrix

I_n - Node-Current Matrix

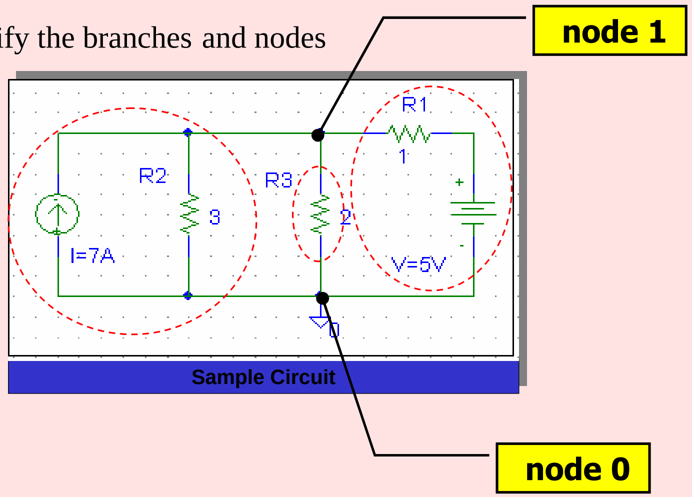
Solutions and solvers

To solve **system of linear equations** we need to use one of the common methods as

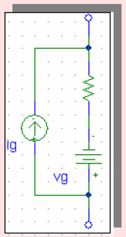
- **Elimination of variables**
- **Row reduction** (Gauss elimination)
- **Cramer's rule**
- **LU decomposition**
- **other methods**

Example

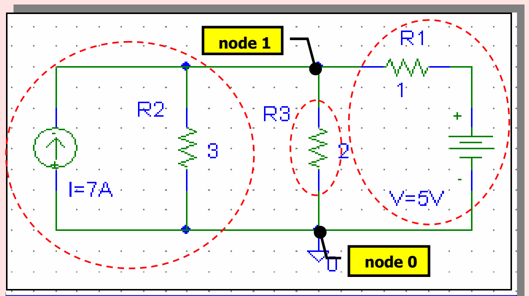
Let's identify the branches and nodes



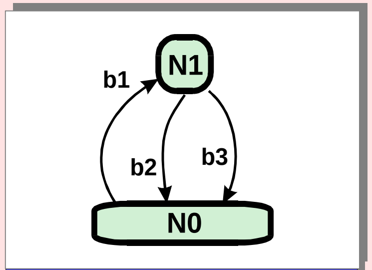
Sample Circuit



Example

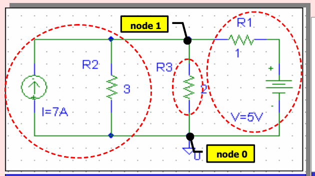


Sample Circuit

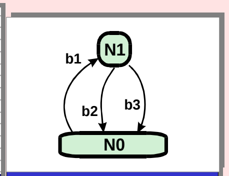


Network Graph

Matrices generation



Sample Circuit



Network Graph

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{Y_b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \quad \mathbf{I_g} = \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix} \quad \mathbf{V_g} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

Putting the numbers into equation

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \quad \mathbf{Y_b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \quad \mathbf{I_g} = \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix} \quad \mathbf{V_g} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A Y_b A^T V_n = A (I_g - Y_b V_g)}$$

$$\begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} V_{n1} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$\begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} V_{n1} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$\begin{bmatrix} -1 & \frac{1}{2} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} V_{n1} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix} - \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$(1 + \tfrac{1}{2} + \tfrac{1}{3}) \begin{bmatrix} V_{n1} \end{bmatrix} = 5 + 7$$

$$\frac{11}{6} V_{n1} = 12$$

$$V_{n1} = 6.545 \text{ volt}$$

Summary

- All electronics circuits might be represented by system of equations
- It is easy to extract the parameters from netlist (.cir file)
- The system of equation is solved using the common methods
- Knowledge about the all nodal voltages allows to find any necessary value (currents, power, etc)

Thank you for your attention